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Estimation of Shear Strength Parameters for C-Phi Soils

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A B S T R A C T

Correlations between engineering parameters are an effective tool in geotechnical engineering practice. The plasticity index and soil cohesion have been shown to be strongly correlated. The same for relative density and friction angle. This study was conducted using 87 soil samples categorized as: low plasticity clay (39 specimens), high plasticity clay (22 specimens), poorly graded sand (6 specimens), and silty sands (20 specimens). The samples undergo direct shear box test (ASTM D 3080) and vane shear test (ASTM D4648) with modification established in this paper. This study intends to compare the results obtained from the vane shear test after adapting it to measure the ultimate torque under different levels of normal stress with the results of direct shear box, to evaluate the validity of the modified vane shear test in obtaining shear strength parameters for c-phi soils. This study also aims at establishing a correlation of cohesion with plasticity index and relative density with angle of friction of soils, using laboratory test data. The paper displays results of the current study and compares them to mathematical relations reported by other researchers in accessible literature.

1. Introduction

Water is an important natural resource. It is an extension of the existence of all living beings and cannot be discarded. The importance of water is not limited to the basic drink of living beings but is due to its chemical properties, which are not found in any other drink. Therefore, every effort must be made to save it from any leakage[1, 2].

Mohr-Coulomb's theory is commonly used for modeling the shear strength of geotechnical materials [1].

According to this theory, soil shear strength varies linearly with applied stress via two shear strength components: the cohesion intercept and the friction angle [2-5].

Soil's effective cohesion (c′) is measured for slope stability and construction foundation suitability. According to Yong and Warkentin (1966) [6], soil effective cohesion is mainly influenced by clay-water interactions, [7-9]. Thus, soil Atterberg limits have an impact on it [10,11].

Correlations between index property parameters and cohesive soil strength properties are commonly used in geotechnical engineering to approximate soil characteristics during preliminary design and validate laboratory test results [12].

Precise estimation of soil shear strength parameters is critical in the construction of many geotechnical structures. shear strength parameters can be determined in either the field or the laboratory. The triaxial compression and direct shear tests are the most used in laboratories to determine cohesion and angle of shearing resistance values [13].

However, experimentally determining the strength parameters is time-consuming and expensive [2,10].

In this study, a little adjustment to the vane shear test has been introduced to determine shear strength parameters under different levels of normal stresses, which will be discussed in detail later in the material and method.

A variety of direct shear tests on the same samples of the modified vane shear test have been conducted and the shear strength parameters for the two tests were compared.

Another aim, this study was conducted to predict shear strength parameters of tested soils based on their plasticity index and relative density.

Relationships between the friction angle with relative density, and cohesion with plasticity index have been proposed. The proposed relationships were compared to other similar relationships found in the literature. The proposed correlations could be beneficial in geotechnical engineering at the feasibility/preliminary design stage for estimating the shear strength parameters based on relative density, friction angle, cohesion, and plasticity index.

Several research work have reported the association between the friction angle and standard penetration number, also standard penetration number with relative density [14-21].

Mujtaba et al. (2018) [22] utilized a data set to correlate relative density and SPT number value was also used to build a relationship between friction angle and relative density. The data was analyzed using regression to identify the relationship between relative density (Dr) and friction angle (φ) determined by direct shear testing. The correlation was calibrated using regression analysis. The final best fit correlation is shown in Figure 1.

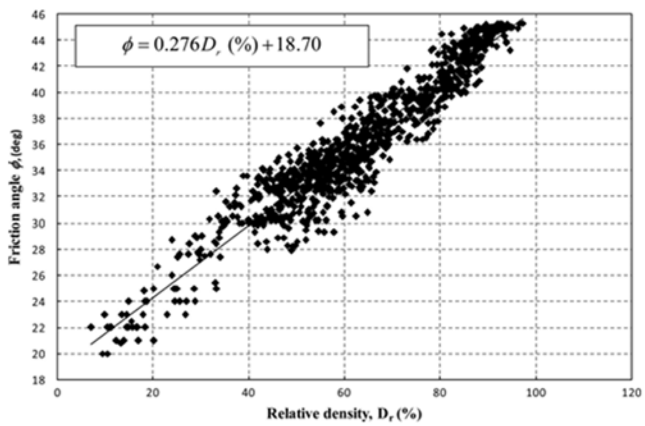

Figure 1: Regression model of φ and D_r Mujtaba et al. (2018) [22]

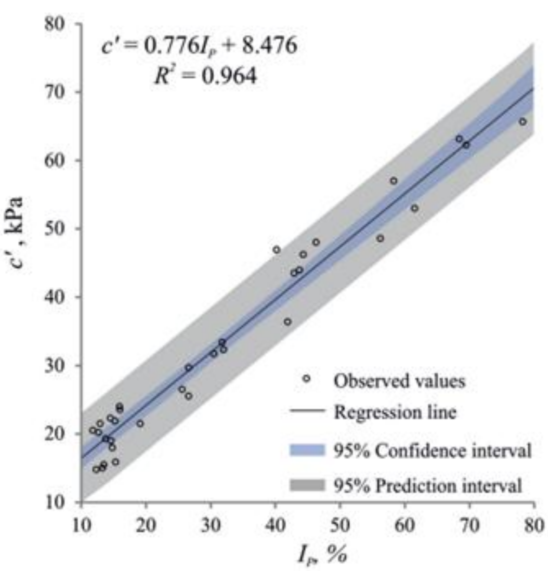

Figure 2: Regression model of c′ and IP Tchakalova and Ivanov (2021) [10]

Tchakalova and Ivanov (2021) [10] used several statistics to investigate the association between c′ and IP. The curve fitting approach was utilized to estimate the regression model.

According to Tchakalova and Ivanov (2021) [10], the linear regression model, displayed in Figure 2, is the best match for estimating c′ of clay soils from their IP.

2. Experimental Work

2.1. Materials and Soil Tests

Samples were collected using two methods: the first was natural sand soil with varying percentages of silt and clay as percent of fines (4-33%), and the second was silty clay samples with a percentage of sand.

A total of 87 soil samples were used to broaden the scope of investigation for c-phi soil studies.

Soil geotechnical index properties were determined, including moisture content, liquid limit, plastic limit, plasticity index, specific gravity, bulk density, and particle size distribution.

The samples have undergone direct shear box test (ASTM D 3080).

Shear resistance envelope was determined by three shear tests done on specimens from the same soil sample at a loading rate of 0.5 mm/min. Each employing a different effective normal stress (30 kPa, 60 kPa, and 71 kPa).

Also, a vane shear test (ASTM D4648) was performed by attaching a loading frame to the top of the soil in the mold and subjecting it to different weights (50 Kg, 100 Kg, and 120 Kg) to determine the ultimate torque under various levels of normal stresses which equal to that undergone by direct shear box test.

The shear strength parameters are then calculated by establishing the relationship between torque and normal stresses. The mathematical equations for calculating shear strength parameters have also been refined.

The next chart (Figure 3) assembles all the investigations that were performed in this research.

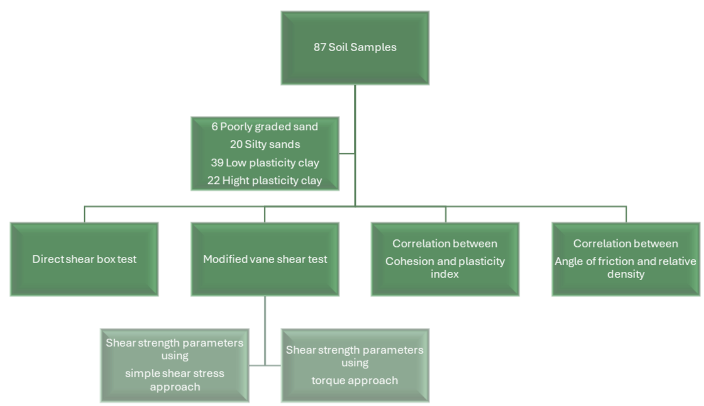

Figure 2: Testing program

2.2. The Model of The Modified Vane Shear Test

A metal disc with a diameter of 15 cm was made with a hole in the center with a diameter of approximately two cm that allowed the vane blade to pass through to the soil. Above the disk, a horizontal steel loading bar was mounted. Loading weights are hung to the loading bar via steel rods hangers at the end of the loading bar as shown in Figure 4. The sample was placed in a mold, and on top of it the disc was minted to apply normal force to the sample during the vane shear test.

The dimensions of mold were chosen in such a way that ratio between mold diameter to the vane shear blade diameter = $\frac{15\ \text{cm}}{1.3\ \text{cm}}$ = 11.5, which ensures that the boundary conditions (friction between steel mold internal surface and adjacent soils) can be ignored, also to facilitate controlling the degree of compaction of the soil in the mold by using the same mold that it is used in standard proctor compaction test.

Figure 4: modified vane shear test

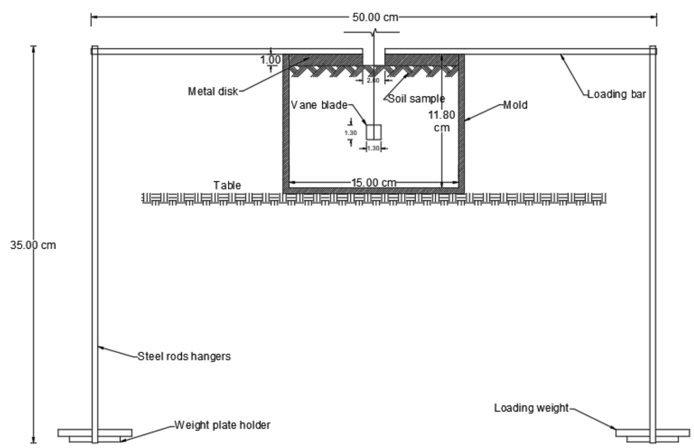

Figure 5: Dimensions of the loading frame and the mold

2.3. Derivation of Proposed Method to Calculate Shear Strength Parameters for Modified Vane Shear Test

Bowles (1996) [23] suggests that the shear stress distribution over the vane border features is as observed in Figure 6. For the cylindrical section, a uniform shear stress is often used (since determining the concentration of stresses would be quite difficult). The stress at the ends is generally defined as the highest shear at D/2 (i.e. along the perimeter).

When a torque T is applied to the soil, it produces two types of shear stresses (vane blade friction with the soil $\tau 1$ in the horizontal top/bottom plans) and soil friction $\tau 2$ (tangential to the cylindrical side surface), as shown in Figure 6, thus torque can be computed by Equation 1:

$$T = \pi \tau_1 \frac{D^3}{6} + \pi \tau_2 \frac{D^2 H}{2} \quad (1)$$

where D and H are diameter in cm and height of vane blade respectively, T is torque in kg.cm, and τ_1 and τ_2 in kg/cm^2 represented as:

$$\tau_1 = c + \sigma_v \tan\varphi \quad (2)$$

$$\tau_2 = c + \sigma_h \tan\varphi = c + k_o \sigma_v \tan\varphi = c + (1 - \sin\varphi)\, \sigma_v \tan\varphi \quad (3)$$

where c is cohesion, φ is angel of friction, σ_h is horizontal stresses (normal to the cylindrical side surface), σ_v is vertical stresses (normal to the top/bottom surfaces), and k_o is coefficient of earth pressure at-rest.

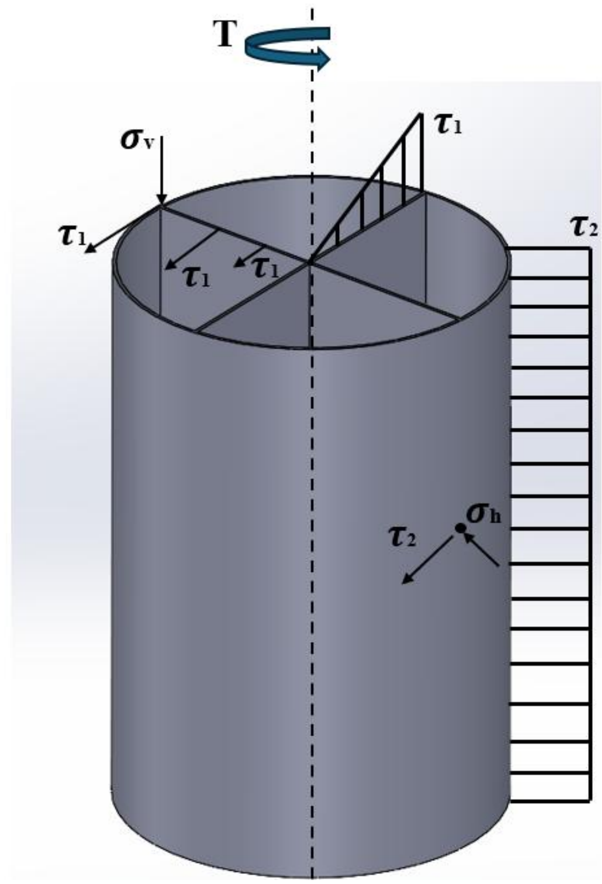

Figure 6: Stresses occurred during the process of modified vane shear test

Vertical stresses calculated from Equation 4:

$$\sigma_v = \frac{N + wieght\ of\ loading\ frame}{A} + \gamma d \quad (4)$$

where N is normal force, A is area of cross section, γ is density of the soil, and d is distance from top of the mold to the middle of vane blade.

The weight of the loading frame equals 3.5 Kg.

Then equation 1 will be:

$$T = \pi\left[(c + \sigma_v (1 - \sin\varphi)\tan\varphi)\frac{D^2 H}{2} + (c + \sigma_v \tan\varphi)\frac{D^3}{6}\right] \quad (5)$$

$$T = \pi\left(\frac{D^2 H}{2} + \frac{D^3}{6}\right) c + \pi \sigma_v \tan\varphi \left[(1 - \sin\varphi)\frac{D^2 H}{2} + \frac{D^3}{6}\right] \quad (6)$$

Units of σ_v and C are in kg/cm^2.

Torque also can be calculated experimentally from Equation 7:

T = (spring constant / 180) × ($\theta_{final} - \theta_{start}$) (7)

where the spring constant of the used vane equals 0.0449 Kg.cm, θ_{final} is final angle of twist, and θ_{start} is start angle of twist.

A straight line can be obtained by establishing a linear relationship between torque and normal stresses. The angle of friction can be calculated from the slope, and the cohesion from the cross section.

$$c = \frac{K}{\pi(\frac{D^2H}{2}+\frac{D^3}{6})} \quad (8)$$

$$\tan\psi = \pi \tan\varphi \; ((1\text{-}\sin\varphi)\frac{D^2H}{2} + \frac{D^3}{6}) \quad (9)$$

where K is cross section and ψ is the slope angle of the relationship between torque and normal stresses.

On the other hand, shear strength parameters can also be calculated using the simplified approach by assuming that the normal stresses along the side surface and top/bottom surfaces are roughly the same, hence:

Shear stress calculated from Equation 10:

$$\tau = \frac{T}{\pi(\frac{D^2H}{2}+\frac{D^3}{6})} \quad (10)$$

$$\tau \; = c \; \sigma_v \; tan \; \varphi \quad (11)$$

where D and H (diameter and height of the blade of used vane) equal 1.3cm. Drawing a linear relationship between the shear and normal stresses, the angle of friction can be calculated from the slope, and the cohesion from the vertical intercept.

3. Results And Discussion

The grain size limits were based on the Unified soil classification system boundaries. As shown in Figure 7, the percentage of passing fines from sieve #200 (0.075mm) is not greater than 50%. This implies that the first type of the soil is coarse grained, whereas the rest of the soil has a finer percentage on sieve #200 (0.075mm) that is greater than 50%, indicating that it is fine grained, as illustrated in Figure 8.

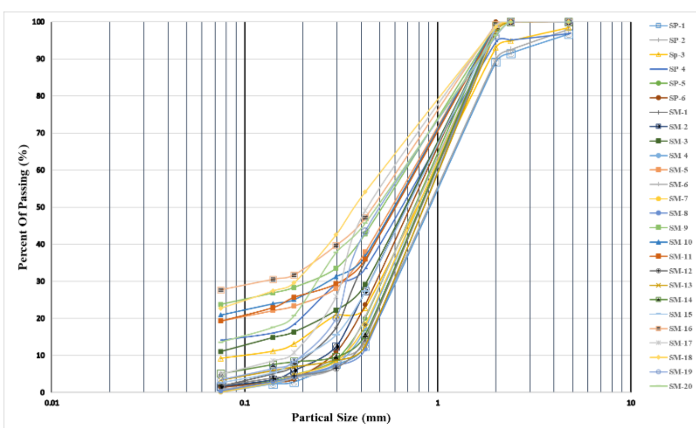

Figure 7: Grain size distribution curve of the coarse-grained samples

The grain size distribution curve of the coarse-grained samples was plotted as shown in Figure 7. The samples are divided into two categories: poorly graded sand (SP) (six specimens), and silty sands (SM) (20 specimens).

Hydrometer analysis was performed on the fines passing through sieve No. 200 for the fine-grained samples, and a grain size distribution curve was plotted as shown in Figure 8.

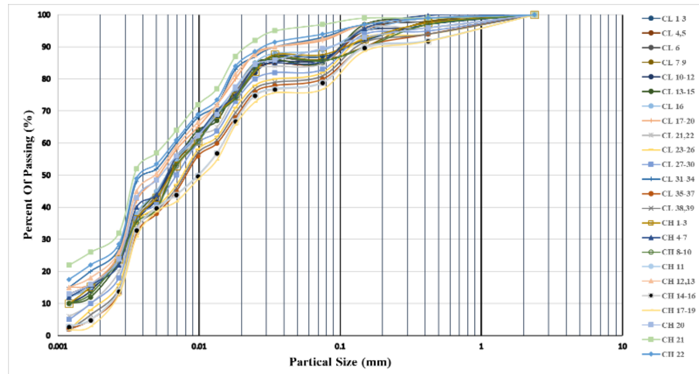

Figure 8: Grain size distribution curve of the fine-grained samples

Plasticity chart was drawn for the fine-grained samples. Figure 9 shows that all samples were above the A line, so all the samples are clay. Twenty-two of the samples had a liquid limit greater than 50%, indicating high plasticity clay (CH), while the remaining 39 samples had a liquid limit less than 50%, indicating low plasticity clay (CL).

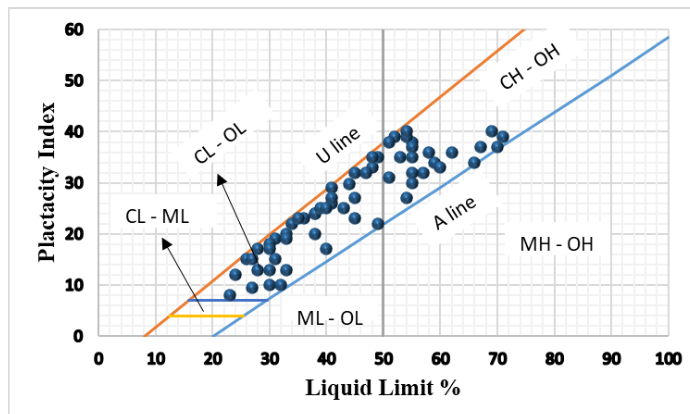

Figure 9: Plasticity chart by the Unified Soil Classification System displays the tested samples

Table 1 summarizes the results of the particle size distribution and presents the uniformity coefficient (Cu) and the coefficient of gradation (Cc) for the four groups.

3.1. Shear strength parameters using direct shear box and the modified vane shear test (torque approach)

The shear strength parameters, cohesion, and angle of friction were determined using the direct shear box and the modified vane shear tests of the same samples.

The results of cohesion and friction angle using the modified vane shear and direct shear box tests were plotted, and coefficient of determination were calculated, as shown in Figure (10,11).

The calculated R2 values range between 0.8895 and 0.8219. It means that the results of the modified vane shear and direct shear box tests are extremely strongly and positively correlated. This also demonstrates the effectiveness of the modified vane shear test approach.

Table 1: particle size distribution, uniformity coefficient and coefficient of gradation for the four groups

Sample Group (USCS)	Gravel (%)	Sand (%)			Fines (%)	C_c	C_u
		Coarse	Medium	Fine			
SP	0-10	57-68	27-38	-	4-5	2.45-3.1	0.82-0.91
SM	0-5	35-67	24-48	-	5-33	2.6-5.3	0.8-1.18
CL	0	0-6	0-7	3-13	80-96	7.4-8.5	0.75-0.84
CH	0-1	1-7	0-5	10-16	76-85	5.7-7.6	0.33-0.94

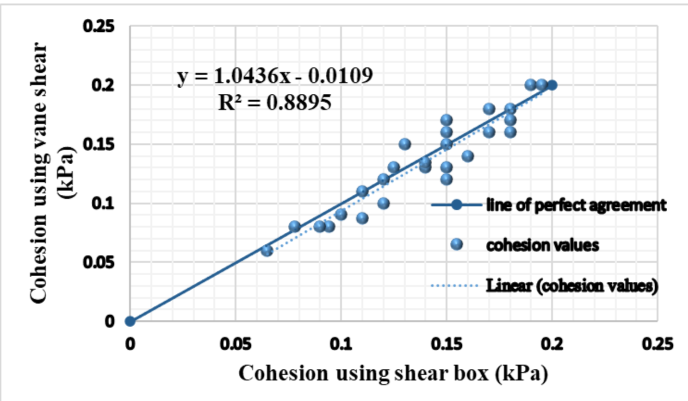

Figure 10: Correlation of shear strength parameters values using the modified vane shear and direct shear box test

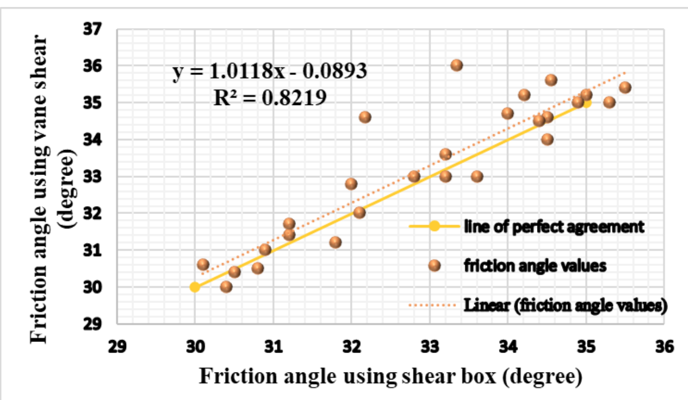

Figure 11: Correlation of shear strength parameters values using the modified vane shear and direct shear box test

3.2. Shear strength parameters using direct shear box and the modified vane shear test (simplified shear stress approach)

The above figures (Figure 10,11) represent the results obtained by torque approach that was proven in this study. thought, Figure 12 uses the simplified shear stress approach.

Figure 12 clearly shows that the correlation doesn't attain a suitable R2 value, demonstrating the validity of torque approach generated by this study and the weakness of the simplified shear stress approach for the modified vane shear test results in calculating friction angle.

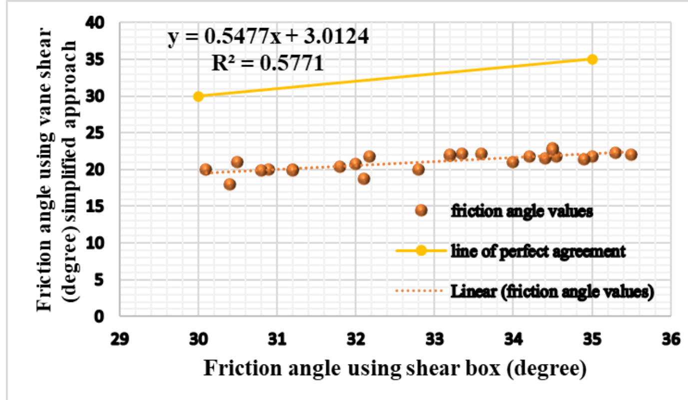

Figure 12: Correlation of friction angle using the modified vane shear by simplified shear stress approach and direct shear box test

This suggests that the simplified approach utilizing modified vane shear estimates a 10–15-degree lower friction angle than the direct shear box values (Figure 12), in contrast to torque approach demonstrated in this study using the modified vane shear with shear box, which is nearly identical as shown in Figure 11.

In terms of cohesion, the simplified approach's estimate is consistent with the shear box's results, as shown in Figure 13. This suggests that the simplified approach gives results close for cohesion only.

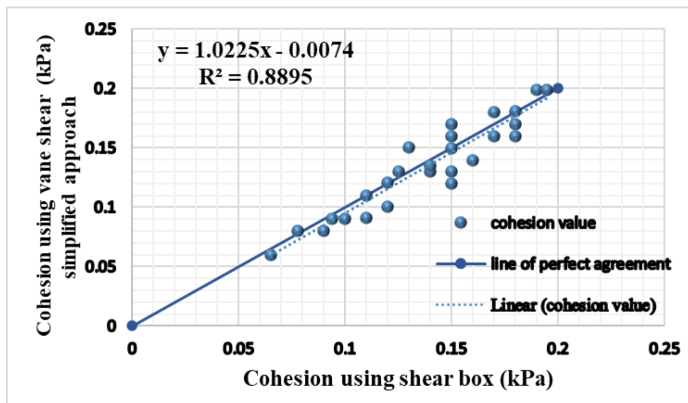

Figure 13 Correlation of cohesion using the modified vane shear by simplified shear stress approach and direct shear box test

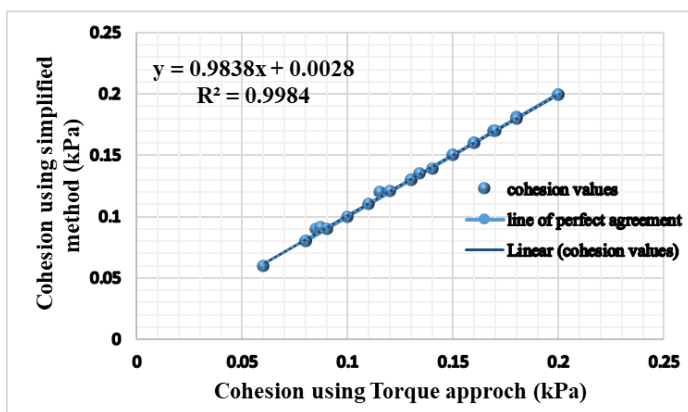

Figure 14: Correlation of cohesion values by torque approach and simplified shear stress approach

3.3. Shear strength parameters using the modified vane shear test (torque and simplified shear stress approach)

The shear strength parameters obtained using torque approach were compared to the simplified approach, as shown in Figure (14,15).

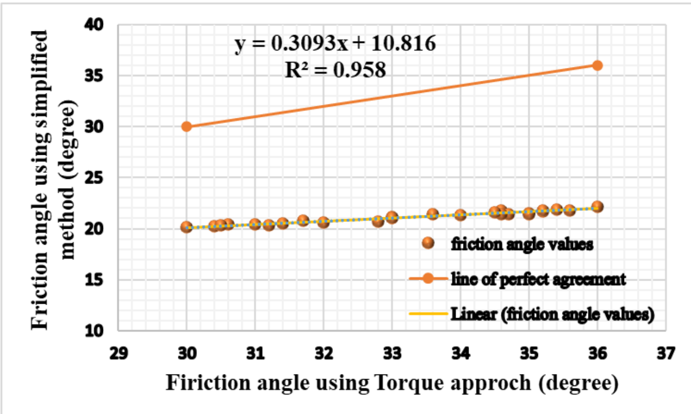

Figure 15: Correlation of friction angle values by torque approach and simplified shear stress approach

The simplified approach using modified vane shear estimates a 10-15-degree lower friction angle than torque approach (Figure 15), as the results from the simplified approach and direct shear box. This is because torque approach values are closer to the values obtained using direct shear box, hence torque approach yields values that are more realistic in calculating friction angle.

The simplified approach's friction angle values are lower than those of the torque approach and direct shear box test values, as it only analyzes vertical stresses and ignores the horizontal stresses on the soil.

3.4. Plasticity index and cohesion correlation

The association between the plasticity index and cohesion was plotted. The graph displayed for the plasticity index and cohesion revealed that the plasticity index has a significant effect on soil cohesion.

There appears to be a reasonably good correlation between the results of the study and what has been collected from the literature. The percentage of plasticity index values in the range of IP = 30-40 demonstrates the slightest variation, indicating the best agreement between the obtained values and the results from the literature.

Figure 16 displays the best estimate. It is as follows:

$C = 0.7702\ I_P + 8.8357$	CL Group	(12)
$C = 0.7186\ I_P + 10.568$	CH Group	(13)

Figure 16 shows that the coefficient of determination R^2 for the CL Group was 0.8097, but for the CH Group's was 0.8109.

3.5. Relative density and friction angle correlation

Furthermore, the graph for relative density and friction angle showed that relative density has a substantial effect on soil friction angle.

There appears to be a reasonably strong correlation between the results of this study and what has been found in the literature. Figure 17 shows the best estimate. It's as follows:

$\Phi = 0.2866\ D_r + 17.968$	CL Group	(14)
$\Phi = 0.2696\ D_r + 17.502$	CH Group	(15)
$\Phi = 0.211\ D_r + 24.07$	SP Group	(16)
$\Phi = 0.1851\ D_r + 24.784$	SM Group	(17)

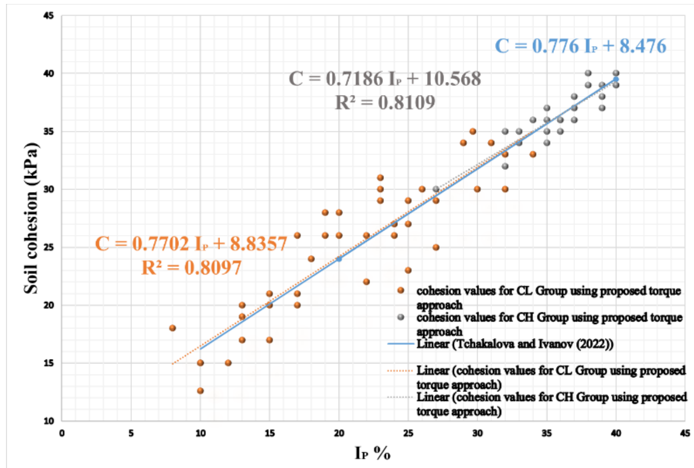

Figure 16: Relationship between c and I_P

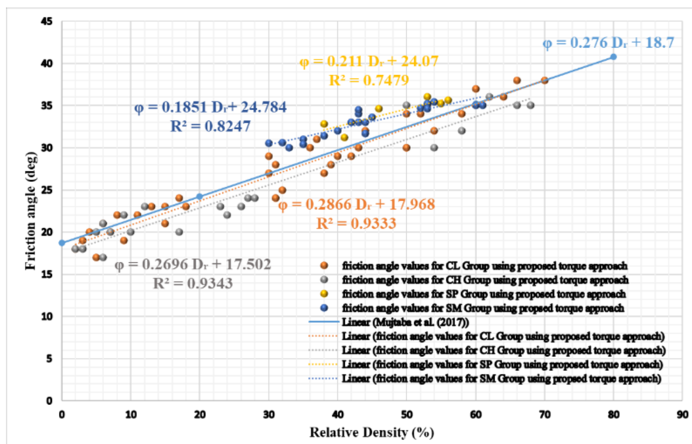

Figure 17: Relationship between φ and Dr

Figure 17 shows that the coefficient of determination R2 for the CL Group was 0.9333, the CH Group's was 0.9343, the SP Group's was 0.7479 and the SM Group's was 0.8247.

Equation 14 and 15 can be used to determine the friction angle based on the model's accuracy.

Table 2 summarizes the results of liquid limit, plastic limit, plasticity index, relative density, and shear strength parameters using the modified vane shear test and direct shear box test for the four groups.

Table 2: (liquid limit, plastic limit, plasticity index, relative density, and shear strength parameters using the modified vane shear test and direct shear box test for the four groups)

Sample Group (USCS)	Liquid Limit (%)	Plastic Limit (%)	Plasticity Index	Relative Density (%)	Modified vane shear test		Direct shear box test	
					Cohesion (kPa)	Friction angle	Cohesion (kPa)	Friction angle
SP	18.9-24.2	NP	NP	38-56	0.08-0.17	31.2-36	0.094-0.15	31.8-34.55
SM	19.8-28.4	18.4-26.1	1.2-3.2	30-61	0.06-0.2	30-35.4	0.065-0.195	30.1-35.5
CL	23-59	11-27	8-34	2-70	13-35	17-38	-	-
CH	47-71	13-33	27-40	2-68	30-40	17-36	-	-

4. Conclusions

In this study, test results from 87 soil samples were used. The samples included 39 low plasticity clays, 22 high plasticity clays, six poorly graded sands, and 20 silty sands.

A suggested approach to determine shear strength parameters for c-phi soils was proposed using vane shear test torque results under different normal stress levels.

The study concludes that:

1. Shear strength parameters using the modified vane shear and direct shear box tests are highly correlated, highlighting the effectiveness of the modified vane shear test approach.
2. A relationship between cohesion and plasticity index was determined, 61 clay soil specimens were tested in the laboratory for index characteristics and cohesion. Correlation analyses were carried out with the plasticity index as the independent variable and cohesion as the dependent variable. The test results indicate that cohesion is connected to the plasticity index. The coefficient of determination R^2 for the CL Group was 0.8097, but for the CH Group's was 0.8109. The resulting equation appears to be applicable to a wide range of clays with I_P values ranging from 9% to 40%.
3. 87 soil specimens were also tested in the laboratory in order to determine if there is a relationship between the angle of friction and relative density for c-phi soils. Correlation analyses were performed with the relative density as the independent variable and the angle of friction as the dependent variable. The test findings revealed that the friction angle can be correlated to relative density. The coefficient of determination R^2 for the CL Group was 0.9333, for the CH Group's was 0.9343, the SP Group's was 0.7479 and the SM Group's was 0.8247. Established correlation allows for the assessment of first estimated values for the soil friction angle.
4. The correlations obtained in the paper show an acceptable level of agreement with the results from existing literature for the two correlations. It appears that the proposed correlation will be an effective assessment tool in the early stages of design.
5. Moreover, the modified vane shear test revealed to be a simple quick test to determine shear strength parameters for c-phi soils.

Conflict of Interest

The authors declare no conflict of interest.

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