



## Parametric Optimization Analysis of Piezoelectric Energy Harvesting from Vehicle Suspension System

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### ARTICLE INFO

*Article history:*

*Received: 30 September 2024*

*Accepted: 4 November 2024*

*Online: 3 March 2025*

**Keywords:**

Energy harvested;

Frequency domain;

Ride comfort;

Piezoelectric.

### ABSTRACT

With the present state of the automobile industry, vehicles need more advanced sensors for control, safety, and operation. As a result of its potential to install low-cost self-powered sensors, vibration energy harvesting has attracted significant scientific interest. In this investigation, a parametric analysis approach is presented for predicting the voltage output and harvested power for two configurations of a two-degree-of-freedom (2DOF) vibration energy harvesting system. A quarter-car suspension model with a piezoelectric element has been chosen for this investigation. The proposed models were mathematically formulated and simulated using MATLAB/Simulink. The analytical technique integrates time domain simulation and frequency response analysis methodologies, thus providing an effective way for designing, and optimizing a 2DOF piezoelectric vibration energy harvester. The energy harvesting performance is evaluated using a comprehensive parametric analysis that includes both design and operational characteristics to determine its effectiveness within the operating frequency range. The findings indicated a greater susceptibility to changes regarding harvested power bandwidth based on the suspension configurations and operating characteristics.

### 1. Introduction

Vibration energy harvesting has garnered significant attention from several fields in recent years due to its potential as an environmentally friendly choice for self-powered wireless sensing devices. In this manner, using piezoelectric materials to gather vibration energy has been extensively investigated. Because of the simplicity of manufacturing and wide frequency range operation, piezoelectric vibration harvesters have been the focus of several studies on obtaining energy[1]. A cantilever beam coupled to a piezoelectric element was the primary focus of the study since it showed potential as an energy source for Micro-electromechanical systems (MEMS) devices [2, 3]. The single degree of freedom (SDOF) piezoelectric energy gathering from vibration performs optimally at a single resonance frequency. As a result, most possible vibration energy sources manifest as arbitrary or changeable frequencies. After much research, the construction of the vibration energy collecting device was fine-tuned and the resonance frequency was adjusted to match the surrounding vibration energy source's frequency. Active structures do not need constant human intervention, as

suggested by Wu and Roundy [4, 5]. These methods increase the average amount of electricity that may be gathered by 30%. Nevertheless, the device's power output is insufficient to operate the resonant frequency tuning mechanism. A multi-DOF piezoelectric energy harvesting from vibration was presented by Shahusz [6]. This device is made up of many single-degree-of-freedom devices connected in series. A unique evaluation of a vehicle suspension model including piezoelectric materials was suggested by Xiao et al. [7] An analytical technique combining time response and frequency responses simulation analysis is beneficial for studying, designing, and optimizing a 2DOF piezoelectric vibration energy harvester. Piezoelectric materials were integrated into the half-vehicle model's front and rear suspension systems by Al-Yafeai et al. [8]. Researchers compared the suggested model's MATLAB/Simulink output to that of the model developed by Xiao et al. [7]model. In comparison to the quarter-car model, the half-car model produced 77% more voltage and 57% more power, according to the data. Numerous studies[9-11] using a quarter-vehicle model demonstrate the impact of road roughness and driving speed on

energy dissipation attributable to tire and suspension properties. Hikmawan et al.[12] suggested a parametric analysis to assess half-car PEH-produced electric power. Vehicle speed and road roughness coefficient were used to examine energy collected. The RMS of produced electric power rises with vehicle velocity and road roughness coefficient. To investigate how various design and operational aspects impact the energy harvesting capabilities of commercial vehicles, Taghavifar et al.[13] developed a standard three-dimensional model that incorporates the suspension's nonlinear damping characteristics. The optimization challenge for a linked ride, road holding, and energy harvesting was addressed [14, 15]. Singh and Satpute [16] used a quarter-car model equipped with an electromagnetic hydraulic shock absorber to mimic its operation. They achieved peak and average speeds of 35 km/h on a smooth city road. This research proposes a parametric analytic approach for evaluating two configurations of a 2DOF quarter car suspension model including piezoelectric materials. The first configuration involves placing the piezoelectric material at the shock tower, which is situated between the chassis and the suspension spring or shock absorber, under a pre-load that has been previously determined. The second option entails integrating the piezoelectric device between the wheel mass and the road's excitation. Monte Carlo simulation will be implemented to conduct sensitivity analysis, with the assumption that the excitation force is uniformly distributed throughout the excitation frequency range [17]. It is assumed that each parameter of the system is normally varied, with the mean value being the original value and the standard deviation being 30% of the mean value for both configurations. The main distinctions between the two systems and the potential for enhancing their energy harvesting performance will be revealed through the evaluation of the parameters. The frequency and time responses simulation results of both configurations have been presented. The energy harvesting performance of vehicles is examined in this research, along with the impacts of various operational and design parameters. Some of these factors are the suspension's damping characteristics and stiffness, the stiffness of the tires, the load placed on the chassis, and the roughness of the road surface.

The present paper is organized as follows: In Section 2, we show a two-degree-of-freedom vehicle suspension combined with a piezoelectric device. Section 3 presents a performance study of two configurations of 2DOF piezoelectric vibration energy harvesting system models. In Section 4, a parametric sensitivity analysis of the proposed two configurations for a two-degree-of-freedom vehicle suspension including a piezoelectric element is presented. Section 5 presents the conclusions

## 2. Modeling of the 2DOF vehicle suspension with a piezoelectric element

In this section, two configurations of a quarter-vehicle suspension with a piezoelectric element are proposed. The difference between the two configurations of the 2DOF

piezoelectric energy harvesting model (PEHM) is the location of the piezoelectric element, their energy harvesting performances are different as shown in Figure 1, which will be characterized using the parameters listed in Table 1. Transmission of vibrations from tire-road contacts via the suspension causes stresses on the piezoelectric element insert, which may be partially turned into electrical energy. According to the vehicle produced by suspension model integrated with piezoelectric elements,  $\mathbf{Z}_o$  represents the excitation displacement;  $\mathbf{M}_1$  stands for the unsprung mass which include the wheel and tire mass of a quarter vehicle;  $\mathbf{M}_2$  is a quarter vehicle's mass;  $\mathbf{K}_1$  is the tire stiffness;  $\mathbf{K}_2$  is the suspension spring stiffness;  $\mathbf{C}$  represents the damping coefficient of suspension system;  $\mathbf{Z}_1$  is the unsprung mass deflection;  $\mathbf{Z}_2$  signifies the body mass displacement;  $\mathbf{V}$  indicates the voltage produced by the piezoelectric insert. In terms of mechanics, the two potential configurations of a quarter vehicle's suspension are defined by:

For a configuration (I)

$$\mathbf{M}_1 \ddot{\mathbf{z}}_1(t) = \alpha \mathbf{V}(t) - \mathbf{k}_2[\mathbf{z}_1(t) - \mathbf{z}_2(t)] - \mathbf{c}[\dot{\mathbf{z}}_1(t) - \dot{\mathbf{z}}_2(t)] + \mathbf{k}_1[\mathbf{z}_o(t) - \mathbf{z}_1(t)] \quad (1)$$

$$\mathbf{M}_2 \ddot{\mathbf{z}}_2(t) = -\mathbf{k}_2[\mathbf{z}_2(t) - \mathbf{z}_1(t)] - \mathbf{c}[\dot{\mathbf{z}}_2(t) - \dot{\mathbf{z}}_1(t)] - \alpha \mathbf{V}(t) \quad (2)$$

$$\alpha[\dot{\mathbf{z}}_2(t) - \dot{\mathbf{z}}_1(t)] = \frac{\mathbf{v}(t)}{\mathbf{R}_1} + \mathbf{c}_o \dot{\mathbf{v}}(t) \quad (3)$$

Where  $\mathbf{R}_1$  represents the cumulative resistance, including both the external load resistance and the internal resistance of the piezoelectric element insert;  $\alpha$  and  $\mathbf{c}_o$  denote the force factor and blocking capacitance of the piezoelectric element, respectively, as stated by

$$\alpha = \frac{eA}{l} \quad \& \quad \mathbf{c}_o = \frac{\epsilon^s A}{l} \quad (4)$$

Where  $\mathbf{A}$  and  $\mathbf{l}$  represent the piezoelectric insert's surface area and thickness, respectively, and  $\mathbf{e}$  and  $\epsilon^s$  stand for the piezoelectric constant and permittivity, respectively.

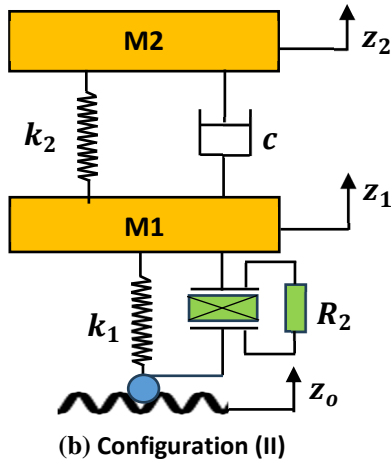
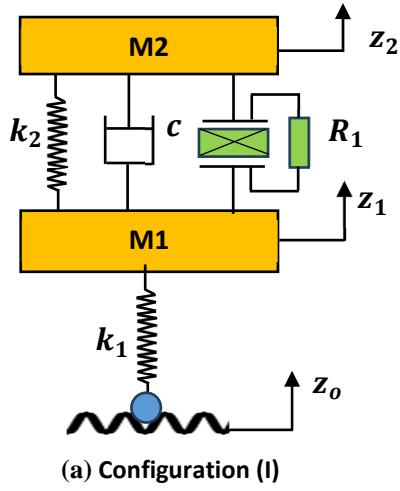


Figure 1: Two configurations of 2DOF piezoelectric energy harvesting model (a) Configuration (I) and (b) Configuration (II)

For a configuration (II)

$$M_1 \ddot{z}_1(t) = \alpha V(t) - k_2[z_1(t) - z_2(t)] - c[\dot{z}_1(t) - \dot{z}_2(t)] + k_1[z_0(t) - z_1(t)] \quad (5)$$

$$M_2 \ddot{z}_2(t) = k_2[z_1(t) - z_2(t)] + c[\dot{z}_1(t) - \dot{z}_2(t)] \quad (6)$$

Table 1: Parameters of the quarter car suspension with a piezoelectric element

Parameter	Meaning	Value	Units
$M_1$	Wheel mass	40	kg
$M_2$	Sprung mass	260	kg
$K_1$	Stiffness of tire	260000	N/m
$K_2$	Spring stiffness of suspension system	26000	N/m
$C$	Damping coefficient of suspension system	1500	N.s/m
$C_0$	Force factor	$1.89 \times 10^{-8}$	
$R_1$	Electrical resistance	30,455.3	$\Omega$
$\alpha$	Piezoelectric element blocking capacitance	$1.52 \times 10^{-3}$	N/V

### 3. Results & Discussions

The performance analysis of the proposed 2DOF piezoelectric vibration energy harvesting system models is investigated based on the quarter car model suspension system. The stimulated acceleration in the mathematical model was represented by a  $9.8 \text{ m/s}^2$  amplitude sinusoidal acceleration that was created using a MATLAB signal-generation module. The output voltage and harvested power were determined. Figure 2 and Figure 3 illustrate the expected output voltage and harvested power using the time domain and frequency domain. When doing a frequency response analysis, it is considered that all other parameters remain constant while the frequency value varies. There are two noticeable resonant peaks: one at 1.54 Hz, which is the mode of suspension bouncing, and the other at 12.3 Hz, which is the mode of wheel hop, or suspension hop as it is more frequently known referenced in [18].

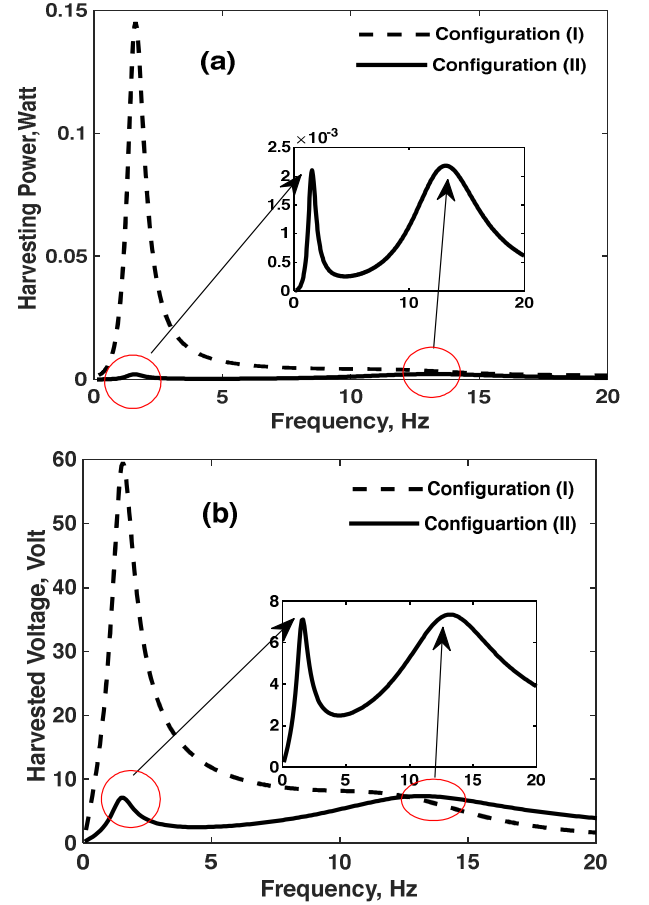


Figure 2: A frequency-domain comparison of the two suggested arrangements' voltage outputs and generated power under acceleration excitation conditions of  $9.81 \text{ m/s}^2$

Both configuration (I) and configuration (II) of the quarter car suspension system are compared concerning the maximum

amplitude of the frequency response curves, which include the output voltage and harvested power.

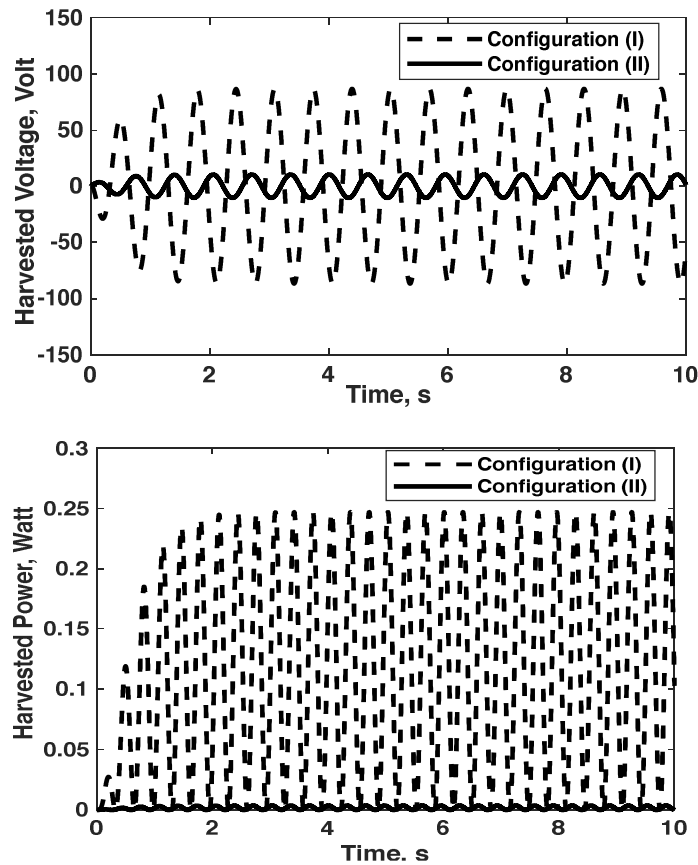


Figure 3: Comparison of output volt and harvested power for the two suggested designs in the time domain under acceleration excitation with an amplitude of  $9.81 \text{ m/s}^2$

In Figure 2a and b, the curves of the harvested power and output voltage in the frequency response indicate that for both the configuration (I) and configuration (II) systems, the peaks of the power and voltage values occur at the first modal resonant frequencies. The voltage output amplitude of the configuration (I) is 6.5 times larger than that of the configuration (II). For identical road excitation amplitudes, the harvestable time response power ranges from 0 to 0.25 W for configuration (I) and from 0 to 0.0035 W for configuration (II) as shown in Figure 3.

Figure 4 (a) and (b) highlight the frequency graphs of the vehicle body acceleration magnitude and the dynamic tire load, normalized by the road excitation displacement amplitude. The two designs provide the same ride comfort and dynamic stability until the stimulation frequency exceeds the natural frequency of the body mass is 1.54 Hz. The first resonance frequency at which the most uncomfortable condition arises should be circumvented. For optimal energy harvesting and riding comfort, it is recommended to focus on the body mass resonant frequency, since it produces a much greater amount of power compared to the wheel mass resonant frequency. The obtained results

concluded that chosen frequency must include the resonant frequency of the vehicle body mass or wheel mass to evaluate the harvested power and output voltage. This effect is not observable when using a restricted frequency range [12].

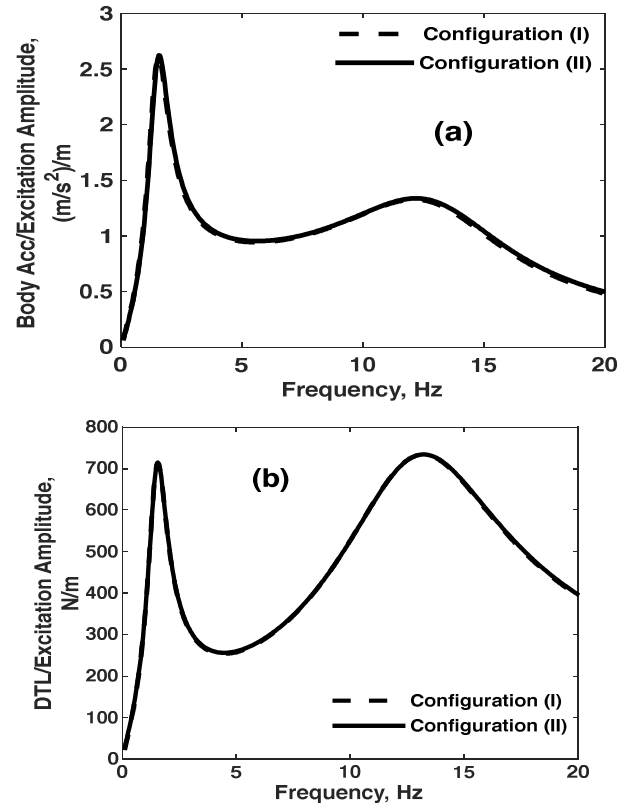


Figure 4: Comparison responses of 2DOF combined with a piezoelectric element at a different location in terms of (a) the body acceleration and (b) the dynamic tire load

#### 4. Parametrical Sensitivity Analysis

Simulation is applied to evaluate the energy harvesting capabilities of a quarter vehicle suspension system paired with a piezoelectric device at a different location and their sensitivities to the parameter change. The excitation frequency in the Monte Carlo model is typically between zero and twenty hertz. The parameters, with their initial values as means and standard deviations of 30% of those means, are considered to follow a normal distribution. These parameters include quarter-car mass  $M_2$ , wheel mass assembly  $M_1$ , suspension spring stiffness  $K_2$ , and suspension damping coefficient  $C$ . Figure 5 to 10 show the normalized harvested power with a random stimulation. The findings can reveal the sensitivity of the energy harvesting and frequency bandwidth to the parameter's variation of both configurations. The blue solid line that divides the gray and black dots in the simulation graphs signifies the harvested power when the parameter is set to its original. It is seen from Figure 5 that when the body mass rises, the collected power magnitude of the bouncing resonant mode will increase, although the bouncing resonant frequency is rarely shifted. In Figure 5, for both configurations of the installation position of the piezoelectric

energy harvester element at the quarter suspension system, the change in vehicle body mass.  $M_2$  limited influences the results in the body mass modal resonate frequency. When the vehicle body mass  $M_2$  is reduced by 30%, the harvestable power at the first modal resonates frequency drops with 0.46% for configuration (I) and 35% for configuration (II). In case of increasing the body mass by 30%, The range of the harvestable power is increased by 0.69% for the configuration (I) and 60% for the configuration (II). For both configurations, there is no change to the energy harvesting response at the second modal resonate frequency.

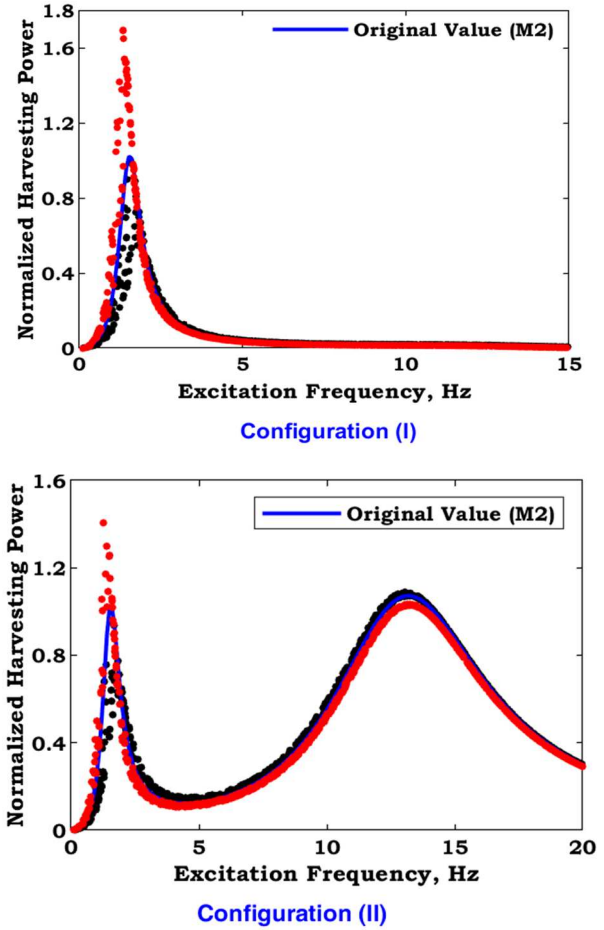


Figure 5: Collected power of quarter vehicle suspension system configurations combined with a piezoelectric element with  $\pm 30\%$  body mass variation versus frequency

Figure 6 shows that when the suspension stiffness  $K_2$  varies, there is a negligible effect on the bandwidths of harvesting frequencies and peak power output for configuration (I). At the first resonant frequency for configuration (II), with 30 % suspension spring stiffness variation, the sensitivity ranges of the harvestable power are 75% for the increasing and 50% for decreasing from its original value. Furthermore, changes in body mass  $M_2$  and spring stiffness  $K_2$  may significantly influence the vehicle's ride comfort; however, it is inadvisable to modify  $M_2$  owing to its negligible impact on energy harvesting performance

and detrimental impacts on vehicle dynamics. In Figure 8, the suspension damping coefficient  $C$  is randomly assigned a mean value and a standard deviation equal to 30% of that value. In all configurations, the modal bounce resonant frequency bandwidth is mostly unaffected by the damping coefficient of the suspension shock absorber, but the amplitude of the bouncing resonant harvested power is very sensitive to it. Also, the ride comfort may be greatly affected by changes in quarter vehicle mass  $M_2$  and suspension stiffness  $K_2$ , therefore it's not an effective strategy to perform with  $K_2$  because of the negative effect on vehicle dynamics and the little influence on energy harvesting performance. Figure 8 shows the results of a randomization of the suspension damping coefficient  $C$  with its nominal value set as the mean and varies with  $\pm 30\%$  of that value. In both configurations, the modal bounce resonating frequency bandwidth is unchanged, but the amplitude of the bouncing resonate harvested power is highly dependent on the suspension shock absorber damping coefficient. The vibration energy harvesting piezoelectric device is best for applications with the lowest suspension dampening coefficient to maximize bouncing resonant collected power.

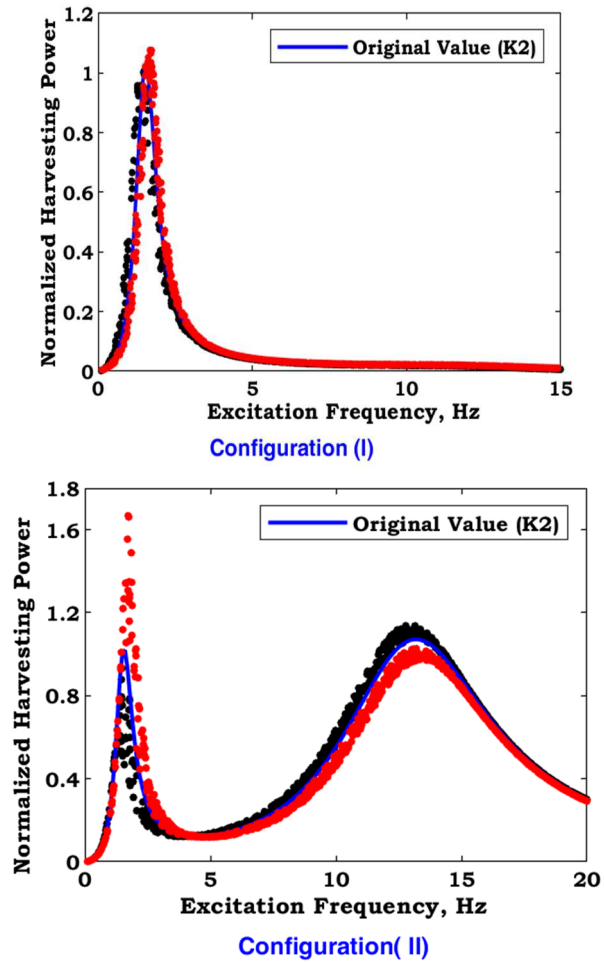


Figure 6: Obtained power of quarter vehicle suspension system configurations combined with a piezoelectric element with  $\pm 30\%$  suspension stiffness variation versus frequency



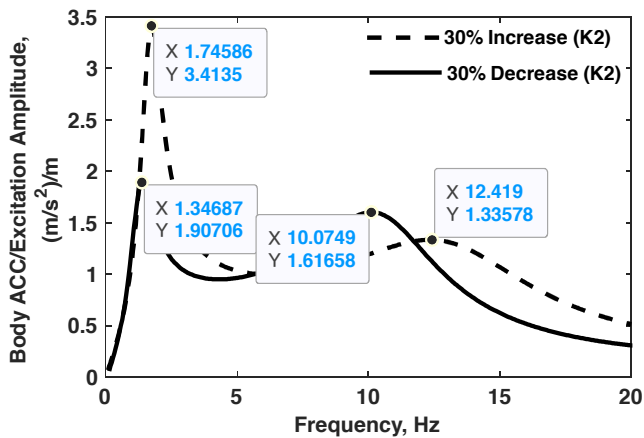
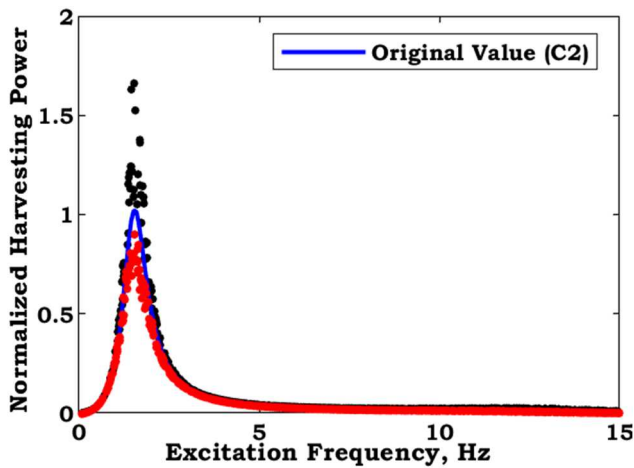
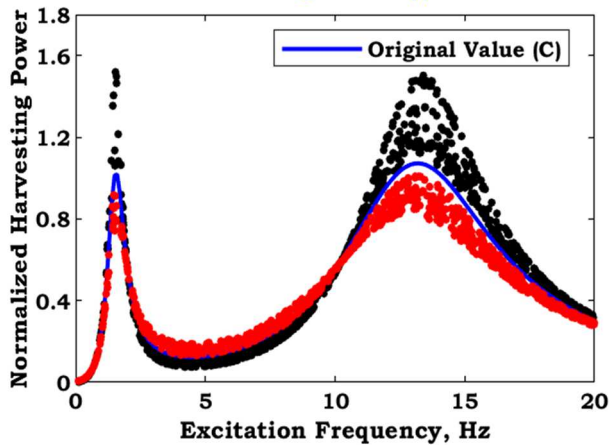


Figure 7: Body acceleration of quarter vehicle suspension system configurations combined with a piezoelectric element with  $\pm 30\%$  suspension stiffness variation versus road excitation frequency.

Figure 9 demonstrates that the amplitude of the harvested power in the bouncing resonant mode is unaffected by changes in the wheel mass  $M_1$  and that the resonant frequency is seldom altered in either setup.



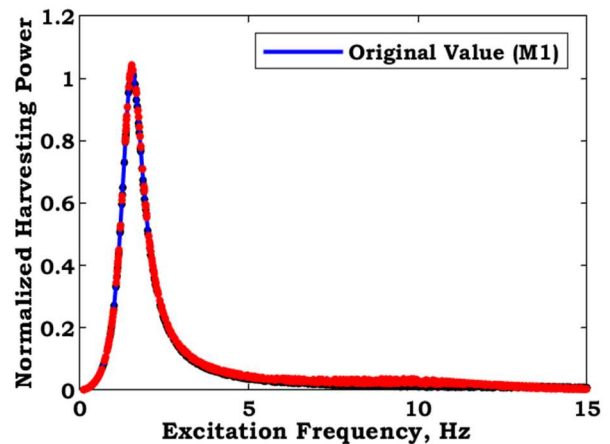
Configuration (I)



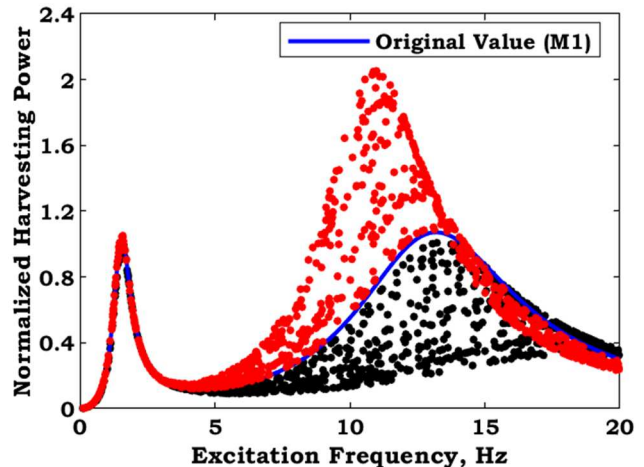
Configuration(II)

Figure 8: Normalized power output for variation of damping coefficient  $C$  with a 30% standard deviation using its nominal value as the mean

A decrease in the hopping resonant frequency and an increase in the power magnitude of the hopping resonant mode are both caused by an increase in wheel-tire mass, and vice versa. For configuration (II) alone, the wheel-tire mass significantly affects the hopping resonant harvesting power magnitudes and frequency bandwidth but not the bouncing frequency and amplitude. Increases in both the sensitivity and bandwidth of the gathered power are caused by changes in the mass of the wheel and tires. Tire stiffness  $K_1$ , displayed in Figure 10, is randomly assigned a value between its initial value and 30% of its standard deviation. The energy harvesting performance for configuration (I) in both the first and second modal resonate frequency with tire stiffness variation is not changed. For configuration (II), the power harvested at bounce frequency resonance is more sensitive to variations in tire stiffness. On the other hand, the second model has a wide range of frequency bandwidth with the change of tire stiffness.

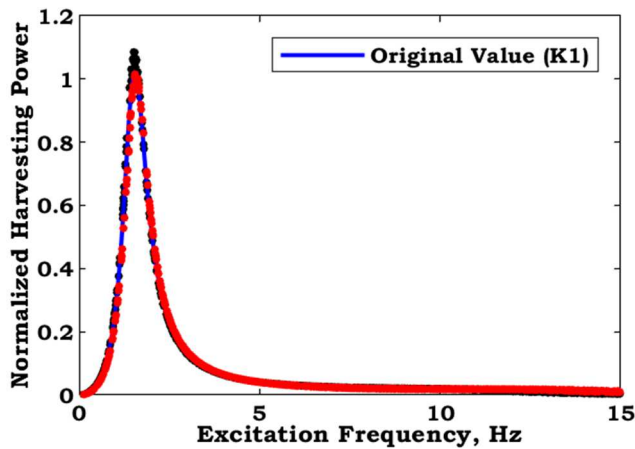


Configuration (I)

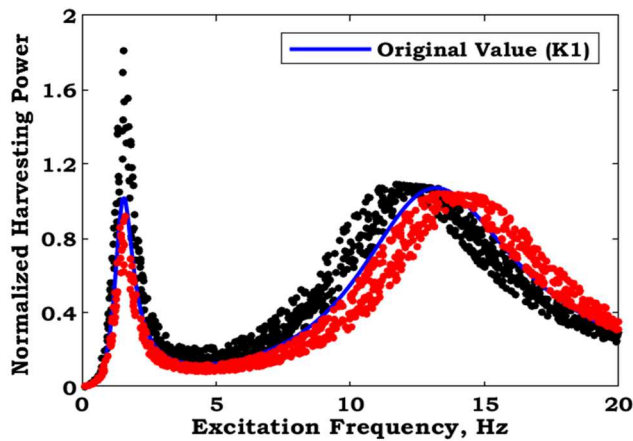


Configuration(II)

Figure 9: Normalized power output for variation of wheel mass  $M_1$  with a 30% standard deviation using its nominal value as the mean for both configurations



Configuration (I)



Configuration( II)

Figure 10: The normalized output power for a variation in wheel tire stiffness ( $K_1$ ) is determined for two configurations by employing the nominal value as the mean with a deviation of 30%

## 5. Conclusions

The study presents a normalized analysis approach to give an accurate and reliable examination of two configurations for the performance of a 2DOF piezoelectric vibration energy harvesting system based on a combination of time domain simulation and frequency response analysis. Based on the findings of the Monte Carlo simulation, indicate that configuration (I) can achieve a greater peak power output than configuration (II). On the other hand, the harvestable energy frequency bandwidth range for configuration (II) is wider than for configuration (I), especially at the second modal resonance frequency. Both configurations are affected by the suspension damping coefficient  $C$  and body mass  $M_2$ , which significantly affect the efficacy of vibration energy harvesting when the peak power output is considered at the first modal resonance frequency. As a result, the suggested theoretical analytical approach may be valuable while developing the 2DOF piezoelectric vibration energy harvester or when fine-tuning the system's setup to get the highest possible gathered power and output voltage. In contrast, because the findings of time domain simulation and frequency response analysis have been confirmed

by each other, the data provided by the hybrid study that combines the two methods may be verified.

## Abbreviations

2DOF	Two Degree of Freedom
SDOF	Single Degree of Freedom
PEHM	Piezoelectric Energy Harvesting Model
MEMS	Micro-electromechanical systems

## Conflict of Interest

The authors declare no conflict of interest.

## References

- [1] Saadon, S. and O. Sidek, *A review of vibration-based MEMS piezoelectric energy harvesters*. Energy Conversion and Management, 2011. **52**(1): p. 500-504 DOI: <https://doi.org/10.1016/j.enconman.2010.07.024>.
- [2] Cook-Chennault, K.A., N. Thambi, and A.M. Sastry, *Powering MEMS portable devices—a review of non-regenerative and regenerative power supply systems with special emphasis on piezoelectric energy harvesting systems*. Smart materials and structures, 2008. **17**(4): p. 043001.
- [3] Choi, W., et al., *Energy harvesting MEMS device based on thin film piezoelectric cantilevers*. Journal of Electroceramics, 2006. **17**: p. 543-548.
- [4] Roundy, S. and Y. Zhang. *Toward self-tuning adaptive vibration-based microgenerators*. in *Smart structures, devices, and systems II*. 2005. SPIE
- [5] Wu, W.-J., et al. *Tunable resonant frequency power harvesting devices*. in *Smart structures and materials 2006: damping and isolation*. 2006. SPIE
- [6] Shahruz, S., *Design of mechanical band-pass filters for energy scavenging*. Journal of sound and vibration, 2006. **292**(3-5): p. 987-998.
- [7] Xiao, H., X. Wang, and S. John, *A dimensionless analysis of a 2DOF piezoelectric vibration energy harvester*. Mechanical Systems and Signal Processing, 2015. **58**: p. 355-375.
- [8] Al-Yafeai, D., T. Darabseh, and A.-H.I. Mourad. *Quarter vs. half car model energy harvesting systems*. in *2019 Advances in Science and Engineering Technology International Conferences (ASET)*. 2019. IEEE
- [9] Xie, X. and Q. Wang, *Energy harvesting from a vehicle suspension system*. Energy, 2015. **86**: p. 385-392.
- [10] Zuo, L. and P.-S. Zhang. *Energy harvesting, ride comfort, and road handling of regenerative vehicle suspensions*. in *Dynamic Systems and Control Conference*. 2011.
- [11] Abdelkareem, M.A.A., et al., *Energy harvesting sensitivity analysis and assessment of the potential power and full car dynamics for different road modes*. Mechanical Systems and Signal Processing, 2018. **110**: p. 307-332 DOI: <https://doi.org/10.1016/j.ymssp.2018.03.009>.
- [12] Hikmawan, M.F., et al., *A Novel Design and Performance Analysis of Piezoelectric Energy Harvester with Application to a Vehicle Suspension System Moving on Uniform Bridges*. International Journal of Technology, 2024. **15**(4).
- [13] Taghavifar, H. and S. Rakheja, *Parametric analysis of the potential of energy harvesting from commercial vehicle suspension system*. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 2019. **233**(11): p. 2687-2700.
- [14] Ataei, M., et al., *Multi-objective optimization of a hybrid electromagnetic suspension system for ride comfort, road holding and regenerated power*. Journal of Vibration and Control, 2017. **23**(5): p. 782-793.
- [15] Mokbel, E.F., et al., *Improving Handling Performance of a Four-Wheel Steering Vehicles Using LQR Controller*. Journal of Advanced Engineering Trends, 2024. **43**(2): p. 509-516.

- [16] Singh, S. and N.V. Satpute, *Design and analysis of energy-harvesting shock absorber with electromagnetic and fluid damping*. Journal of Mechanical Science and Technology, 2015. **29**: p. 1591-1605.
- [17] Hassan, M.A., et al., *A monte carlo parametric sensitivity analysis of automobile handling, comfort, and stability*. Shock and Vibration, 2021. **2021**(1): p. 6638965.
- [18] Yagiz, N., Y. Hacıoglu, and Y. Taskin, *Fuzzy sliding-mode control of active suspensions*. IEEE Transactions on Industrial Electronics, 2008. **55**(11): p. 3883-3890 DOI: 10.1109/TIE.2008.924912.