Journal of Advanced Engineering Trends
ISSN : 2682 - 2091

Vol. 44, No. 2, July 2025

A Moho Model for Africa

Hussein A. Abd-Elmotaal[1], Kurt Seitz[2], Norbert Kuehtreiber[3], Thomas Grombein[2], Bernhard Heck[2] and Hansjoerg Kutterer[2]

[1] *Civil Engineering Dep., Faculty of Engineering, Minia University, Minia, Egypt*
[2] *Geodetic Institute, Karlsruhe Institute of Technology, Karlsruhe, Germany*
[3] *Institute of Geodesy, Graz University of Technology, Graz, Austria*

* *Corresponding author(s) E-mail:* hussein.abdelmotaal@gmail.com

A R T I C L E I N F O

Article history:
Received: 3 April 2025
Accepted: 22 May 2025
Online: 1 June 2025

Keywords:
Moho depth
Africa
geoid
gravity
least-squares prediction

A B S T R A C T

The Mohorovičić´ discontinuity (Moho depth) is important information regarding the Earth's interior. It is used in several applications in geodesy and geophysics. Recent advances in Moho models, which integrate seismic, gravity, and satellite data, provide new opportunities for several geodetic and geophysical applications, e.g., enhancing the geoid determination accuracy, understanding regional geodynamics and plate motion, as well as studying the tectonic motion and earthquake monitoring. Several Moho models were tested, ultimately leading to the creation of a composite model for Africa. This model was derived using least-squares interpolation technique. In order to perform the least-squares interpolation technique, a trend surface has been removed from all source Moho data to satisfy the necessarily condition of having centered interpolated field, i.e., a field with zero mean. A smart approach for fitting the covariance function has been implemented. The removed trend surface has been restored after the interpolation process yielding the $3' \times 3'$ AFRMoho25 Moho model for Africa.

1. Introduction

Africa, the world's second largest continent, suffers from lack of gravity data in some regions. This represents the main challenge facing most of the geodetic and geophysical applications in the continent.

As simple interpolation of the existing gravity data does not add new information at the large data gaps, therefore the current investigation suggests the usage of Moho models in order to better estimate the gravity values at the African large data gaps, which serves for various geodetic/geophysical applications. The ultimate goal of this paper is the creation of a composite Moho model for Africa.

Moho models now exist, where a combination of seismic, gravity and satellite data takes place (see, e.g., Sjöberg and Abrehdary, 2022; Ye et al, 2017; Reguzzoni and Sampietro, 2015; Reguzzoni et al, 2013; Laske et al, 2013; Čadek and Martinec, 1991; Geiss, 1978). The Moho information provides new opportunities for several geodetic and geophysical applications, e.g., enhancing the geoid determination accuracy. The following Moho models were used in the current research:

- Čadek and Martinec (Čadek and Martinec, 1991)
- GEMMA (Reguzzoni and Sampietro, 2015)
- CRUST 1.0 (Laske et al, 2013)
- MOHV21 (Sjöberg and Abrehdary, 2022)

A Moho model for Africa has been created using the above four models as input data. These data have been de-trended to allow the usage of the least-squares interpolation technique, i.e., the used data are centered (having nearly zero average). The de-trended Moho data have been given different weights in the interpolation process depending on their resolution and frequency content. Then the removed trend has been restored after the interpolation process creating the AFRMoho25 Moho model for Africa. This work is conducted as an important activity of the International Association of Geodesy (IAG) Sub-Commission on gravity and geoid in Africa.

2. Moho Data

The Moho models used are described in the following subsections.

2.1. Cadek and Martinec

Čadek and Martinec (1991) have compiled crustal- mantle boundary depths from various sources and expressed the crustal thickness in a spherical harmonic expansion up to a maximum degree (personal communication with the first author). The crustal thickness can be given by

$$h(\theta,\lambda) = \sum_{n=0}^{n_{\max}} \sum_{m=0}^{n} \left(\bar{h}_{nm}^{c} \cos m\lambda + \bar{h}_{nm}^{s} \sin m\lambda\right) \bar{P}_{nm}(\cos\theta) \tag{1}$$

where $\bar{h}^c_{nm}, \bar{h}^s_{nm}$ are the fully normalized harmonic coefficients given by (Čadek and Martinec, 1991), θ is the polar distance, λ is the geodetic longitude, and $\bar{P}_{nm}(\cos\theta)$ denotes the fully normalized associated Legendre functions. The polar distance θ can simply be expressed in terms of the geocentric latitude ψ as:

$$\theta = 90^\circ - \psi, \qquad (2)$$

where ψ is related to the geodetic latitude ϕ through the following expression (Torge, 1980, p. 50):

$$\tan\psi = (1-f)^2 \tan\phi, \qquad (3)$$

where f is the flattening of the Earth's reference ellipsoid.

Figure 1 shows the Čadek and Martinec Moho depths for Africa computed by (1) complete to the maximum degree $n_{\max} = 70$ on a 0.5º × 0.5º grid. The Čadek and Martinec Moho depths range between 3.24 and 52.59 km with an average of about 21.85 km and a standard deviation of 13.18 km. Figure 1 shows long to medium wavelength structure based on the used harmonic model.

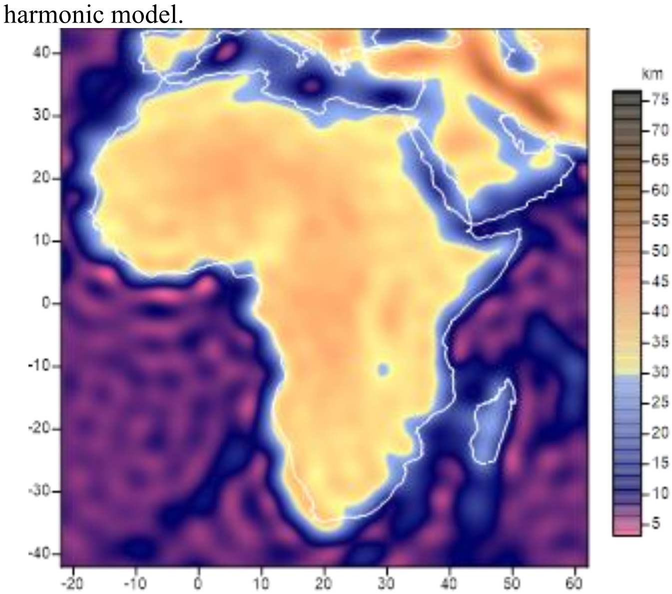

Figure 1: Čadek and Martinec Moho depths for Africa using Čadek and Martinec (1991) harmonic coefficients complete to the maximum degree

2.2. GEMMA Model

Reguzzoni and Sampietro (2015) have generated GEMMA model based on inverting GOCE satellite gravity data. Figure 2 shows the GEMMA Moho depths for Africa, which is available on a global 0.5º × 0.5º grid. GEMMA Moho depths range between 6.04 and 76.71 km with an average of about 24.14 km and a standard deviation of 11.57 km. The GEMMA model shows more feature contents of the Moho, especially on land. The values of the Moho depths on oceans are significantly larger than those of the Čadek and Martinec Moho depths. Also on land, the GEMMA values are much higher, especially on mountainous areas. This may result directly from inverting the GOCE gravity anomalies, which are correlated to topography.

2.3. CRUST1.0 Model

Laske et al (2013) have developed the CRUST1.0 Moho model, which is based on 1º × 1º averages of a recently updated database of crustal thickness data from active source seismic studies as well as from receiver function studies. In areas where such constraints are still missing, for example in Antarctica, crustal thicknesses are estimated using gravity constraints.

Figure 3 shows the CRUST1.0 Moho depths for Africa. CRUST1.0 Moho depths range between 8.27 and 50.15 km with an average of about 25.22 km and a standard deviation of 12.46 km. Figure 3 shows a coarser pixel structure than one degree, which signalizes the basis of building up this Moho model. It also shows a long to medium frequency content of the Moho depths.

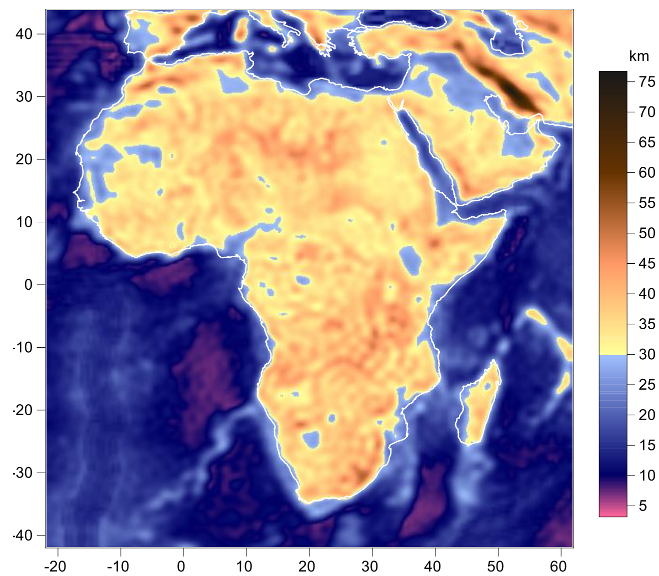

Figure 2: GEMMA Moho depths for Africa (Reguzzoni and Sampietro, 2015).

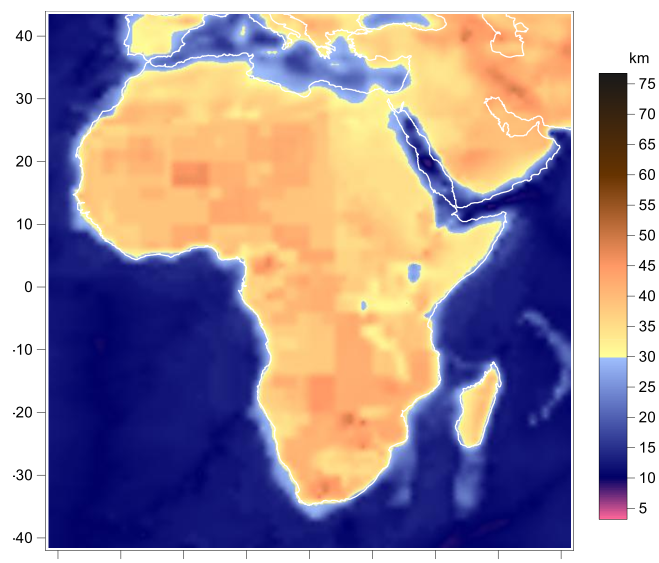

Figure 3: CRUST1.0 Moho depths for Africa (Laske et al, 2013).

2.4. MOHV21 Model

Sjöberg and Abrehdary (2022) have developed the MOHV21 Moho model based on an optimal combination of five global seismic and gravimetric-isostatic models of Moho depth by a weighted least squares approach at a resolution of $1^{\circ} \times 1^{\circ}$.

Figure 4 shows the MOHV21 Moho depths for Africa. MOHV21 Moho depths range between 8.02 and 50.51 km with an average of about 25.73 km and a standard deviation of 12.89 km. Figure 4 shows again a long to medium frequency content of the Moho depths. Comparing Figures 3 and 4 indicates clear similarities between the CRUST1.0 and MOHV21 Moho models.

3. Creating the Moho Model for Africa Employing Least-Squares Prediction Technique

In order to apply the least-squares prediction technique (Moritz, 1980), the expectation (mean) of the predicted field should be nearly equal to zero, i.e.,

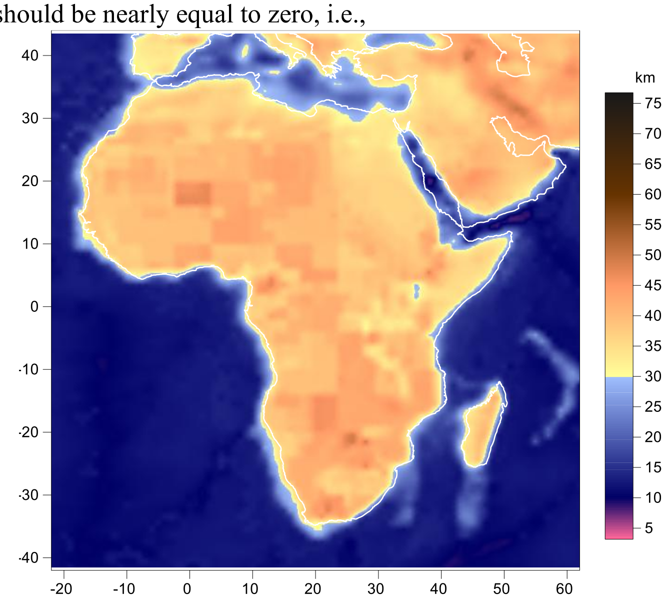

Figure 4: MOHV21 Moho depths for Africa (Sjöberg and Abrehdary, 2022)

$$E\{\cdot\} \cong \text{zero} \quad (4)$$

This is, of course, not the case for the Moho depths, as they are always greater than zero.

In order to overcome this problem, a remove- interpolate-restore scheme has been suggested. It is described in the following steps:

- Remove a trend surface from the four available Moho data sets. The residual fields then have nearly a zero mean satisfying (4).
- Perform the interpolation whose data are the residual fields resulting from the previous step.
- Restore the trend surface to create the Moho model for Africa.

As for the trend surface, a polynomial surface has been employed. Least-squares regression technique has been used to fit the polynomial trend surface to the Moho data. Several trend surfaces have been tested. A surface polynomial of the 8th degree has proved to give the best trend surface in terms of the residual field.

Table 1. Statistics of the Moho residuals for Africa after removing a polynomial trend surface of the 8th degree. Units are in [km]

Residual Moho model	min.	max.	mean	std
Čadek and Martinec	−22.24	19.63	−1.18	5.46
GEMMA	−24.73	38.70	1.20	6.44
CRUST1.0	−22.15	24.85	2.08	5.16
MOHV21	−22.79	25.45	2.63	5.38
Integrated field of the four residual models	-24.73	38.70	0.47	6.01

Table 1 illustrates the statistics of the Moho residuals for Africa after removing a surface polynomial trend surface of the 8th degree. It shows that the residual Moho fields satisfy the necessary condition to apply the least-squares prediction technique.

The used least-squares interpolation technique employs the generalized covariance model of Hirvonen (Moritz, 1980)

$$C(s) = \frac{C_o}{(1 + A^2 s^2)^p} \quad (5)$$

with

$$A = \frac{1}{\xi}\left(2^{\frac{1}{p}} - 1\right)^{\frac{1}{2}}, \quad (6)$$

where C_o and ξ are the empirically determined variance and correlation length, respectively, and s is the spherical distance between the pair of points under consideration. The dimensionless curvature parameter χ is related to the curvature κ of the covariance function at $s = 0$ by (Kraiger, 1988)

$$\chi = \frac{\kappa \xi^2}{C_o}. \quad (7)$$

Therefore, it affects the values of the covariances near the origin. The curvature parameter χ is related to the parameter p as (Moritz, 1976; Abd-Elmotaal, 1992)

$$\chi = 2p\left(2^{\frac{1}{p}} - 1\right). \quad (8)$$

The estimation of the parameter p results from the fitting of the empirically determined covariance function by employing a least-squares regression algorithm developed by Abd-Elmotaal and Kühtreiber (2016). A value of $p = 0.492$ has been estimated. The values of the empirically determined variance C_o and correlation length ξ for the empirical covariance function of the integrated residual Moho field are as follows:

$$C_o = 33.03 \text{ km}^2, \quad \xi = 203.13 \text{ km}. \quad (9)$$

Figure 5 illustrates the fitting of the empirically determined covariance function performed by the least-squares regression algorithm (Abd-Elmotaal and Kühtreiber, 2016). The very good fitting of the empirically determined covariance function is evident.

The Moho data models have different structures and frequency contents as shown in Figs. 1 to 4. Therefore, it has been decided to give them different weights in the least-squares prediction technique depending on their resolution and frequency content. Table 2 shows the standard deviation given for each of the Moho residual sets. The least-squares interpolation process has been carried out on a uniform grid of 3′ × 3′ in order to generate a Moho model for Africa with that resolution.

After the least-squares process has taken place, the removed surface polynomial trend has been restored generating the AFRMoho25 3′ × 3′ developed Moho model for Africa. It is shown in Fig. 6. The Moho depths of the AFRMoho25 developed model for Africa range between 5.55 and 62.04 km with an average of about 23.17 km and a standard deviation of 12.04 km.

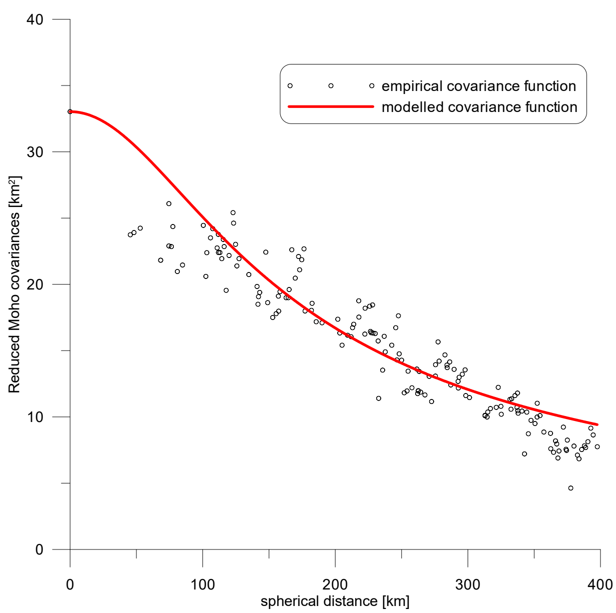

Figure 5: Fitting of the empirically determined covariance function using least-squares regression algorithm (Abd-Elmotaal and Kühtreiber, 2016)

Table 2: Standard deviation given for each of the Moho residuals in the least-squares prediction technique

Residual Moho model	assigned standard deviation [km]
Čadek and Martinec	2.5
GEMMA	2.5
CRUST1.0	5.0
MOHV21	5.0

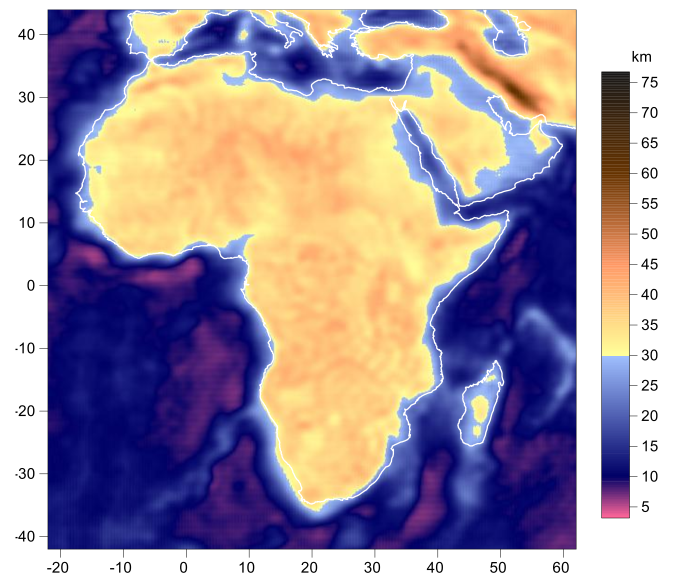

Figure 6: The AFRMoho25 3′ × 3′ developed Moho model for Africa

In order to investigate the correlation between the Moho depths of the AFRMoho25 model and the topography, the AFH16M03 3′ × 3′ Digital Terrain Model (DTM) for Africa (Abd-Elmotaal et al, 2017) is utilized and shown in Fig. 7. The heights of the AFH16M03 DTM range between –7630 and 5024 m with an average of 1623 m and a standard deviation of about 2406 m. Over the oceans, the model heights are related to bathymetry. A comparison of Figs. 6 and 7 shows that the Moho depths of the AFRMoho25 model are generally correlated with the topography.

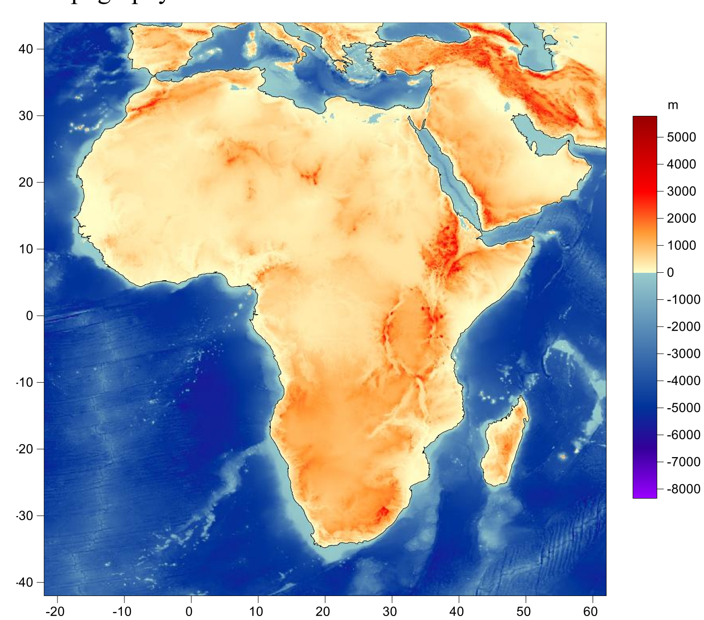

Figure 7: The 3′ × 3′ AFH16M03 DTM for Africa (Abd- Elmotaal et al, 2017). Units in [m].

4. Evaluation

As the Moho depths of the AFRMoho25 developed model for Africa show a significant correlation to the topography, it has been chosen to compare them with the Moho depths generated by using the plate loading theory (Abd-Elmotaal, 1993).

To generate the Moho depths by employing the plate loading theory, several parameter sets have been extensively tested. The following parameter set proved to give the nearest Moho depths compared to the AFRMoho25 Moho depths developed in Sec. 3:

$$
\begin{aligned}
T_o &= 30\,\mathrm{km}\,, \\
\rho_o &= 2.30\,\mathrm{g/cm^3}\,, \\
\Delta\rho &= 0.27\,\mathrm{g/cm^3}\,, \\
l &= 70\,\mathrm{km}\,,
\end{aligned}
\tag{10}
$$

where T_o denotes the normal crustal thickness, ρ_o is the density of the Earth's crust, $\Delta\rho$ is the density contrast between the lower crust and the upper mantle, and l is the degree of regionality, given by Vening Meinesz (1940)

$$
l = \sqrt[4]{\frac{D}{g(\rho_1 - \rho_o)}}\,, \tag{11}
$$

where g is the gravity, D is the cylindrical rigidity of the Earth's crust, and ρ_1 is the density of the mantle.

Figure 8 shows the 3′ × 3′ Moho depths for Africa generated by the plate loading theory using the above specified parameter set (10). These Moho depths range between 3.20 and 48.99 km with an average of about 23.64 km and a standard deviation of 12.22 km. Figure 8 shows smooth Moho depths with obvious correlation to the topography.

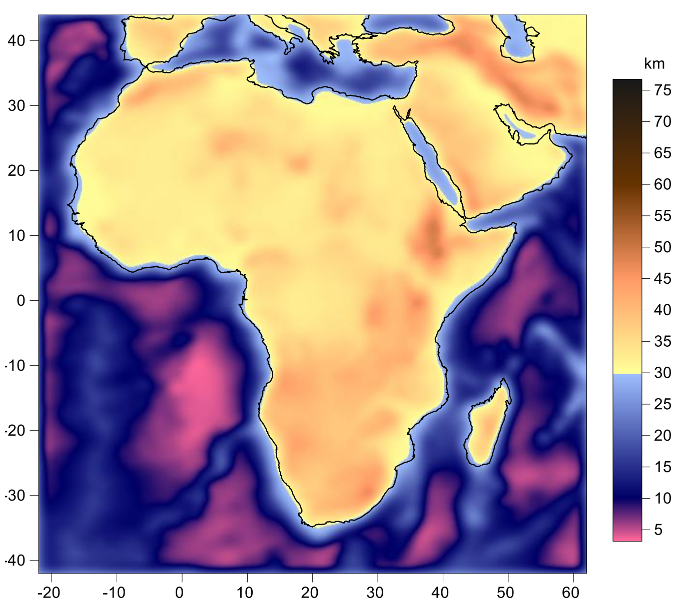

Figure 8: 3′ × 3′ Moho depths for Africa generated by the plate floating theory based on the AFH16M03 DTM using the parameter set (10)

Figure 9 shows the difference between the AFRMoho25 developed Moho model and the Moho depths generated by the plate loading theory for Africa. These differences range between –15.64 and 22.97 km with an average of about 0.69 km and a standard deviation of 3.80 km. The white pattern in Fig. 9 indicates differences below 5 km in magnitude. Figure 9 shows that the AFRMoho25 developed Moho depths for Africa within the current investigation match to a great extent with the Moho depths generated by the most realistic isostatic hypothesis. However, larger differences occur at the high lands of Ethiopia and at the Red Sea.

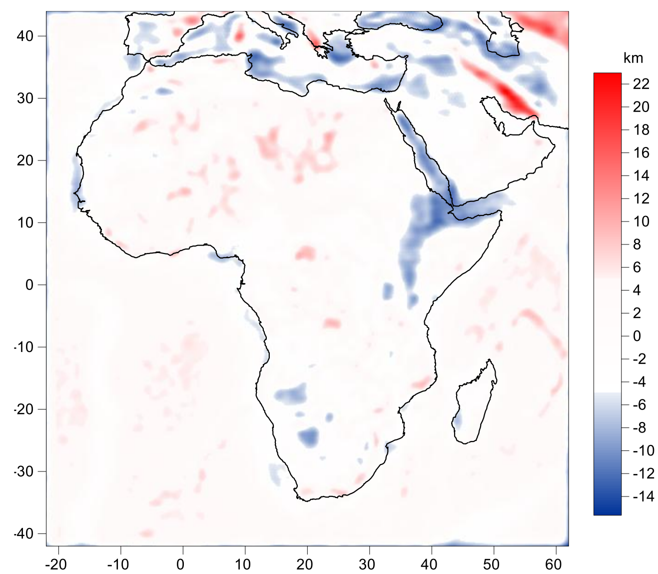

Figure 9: 3 Difference between the AFRMoho25 developed Moho model and the Moho depths generated by the plate floating theory for Africa

5. Conclusions

Moho depths play an important role in many geodetic and geophysical applications, such as the geoid determination, defining the height reference system, studying the plate tectonics, etc. The AFRMoho25 Moho model for Africa has been successfully established from the available data sources using the least-squares prediction technique within the remove-interpolate-restore scheme. The AFRMoho25 Moho model has been compared with the Moho depths generated using the most realistic isostatic theory, the plate loading theory. This comparison shows generally good agreement except at the red Sea and the high lands of Ethiopia. This might signalize that the density of the crust there is far from the assumption of a constant density appearing in (10). However, these discrepancies deserve a deeper geophysical investigation.

The AFRMoho25 developed Moho model for Africa in the current investigation will be used in the generation of the updated gravity database for Africa, which is necessary for the geoid computation of the continent. This is, however, a crucial task needed by the International Association of Geodesy.

Conflict of Interest

The authors declare no conflict of interest.

Acknowledgement

The authors would like to thank the editor of the current manuscript and the anonymous reviewers for their efforts and useful comments.

References

[1] Abd-Elmotaal, HA (1992) Statistical behaviour of the free-air, Bouguer and isostatic anomalies in Austria. Bulletin Géodésique 66(4):325–335, DOI 10.1007/BF00807417.

[2] Abd-Elmotaal, HA (1993) Vening Meinesz moho depths: traditional, exact and approximated. Manuscripta geodaetica 18(4):171–181, DOI https://doi.org/10.1007/BF036 55311.

[3] Abd-Elmotaal, HA and Kühtreiber, N (2016) Effect of the curvature parameter on least-squares prediction within poor data coverage: Case study for Africa. Geophysical Research Abstracts 18, URL https://meetingorganizer.copernicus.org/EGU2016/EGU2016-271.pdf.

[4] Abd-Elmotaal, HA, Makhloof, A, Abd-Elbaky M and Ashry, M (2017) The African 3″ × 3″ DTM and its validation. International Association of Geodesy Symposia 148:79–85, DOI https://doi.org/10.1007/1345_2017_19.

[5] Čadek, O and Martinec, Z (1991) Spherical harmonic expansion of the earths crustal thickness up to degree and order 30. Studia Geophysica et Geodaetica 35:151–165, DOI https://doi.org/10.1007/BF01614063.

[6] Geiss, E (1978) A new compilation of crustal thickness data for the Mediterranean area. Annales Geophysicae 5B(6):623–630.

[7] Kraiger, G (1988) Influence of the curvature parameter on least-squares prediction. Manuscripta Geodaetica 13(3):164–171, DOI https://doi.org/10.1007/BF03655244.

[8] Laske G, Masters, G, Ma, Z and Pasyanos M (2013) Update on CRUST1.0 – A 1-degree global model of earth's crust. Geophysical Research Abstracts 15, URL https://meetingorganizer.copernicus.org/EGU2013/ EGU2013-2658.pdf.

[9] Moritz, H (1976) Covariance functions in least-squares collocation. Ohio State University, Department of Geodetic Science and Surveying, Rep 240, URL https://earthsciences.osu.edu/sites/earthsciences.osu. edu/files/report-240.pdf.

[10] Moritz, H (1980) Advanced Physical Geodesy. Wichmann, Karlsruhe.

[11] Reguzzoni, M, Sampietro, D (2015) GEMMA: An Earth crustal model based on GOCE satellite data. International Journal of Applied Earth Observation and Geoinformation 35:31–43, DOI https://doi.org/10.1016/j.jag.2014.04.002.

[12] Reguzzoni, M, Sampietro, D and Sansò F (2013) Global Moho from the combination of the CRUST2.0 model and GOCE data. Geophysical Journal International 195:222–237, DOI https://doi.org/10.1093/gji/ggt247.

[13] Sjöberg, LE and Abrehdary, M (2022) MOHV21: a least squares combination of five global moho depth models. Journal of Geodesy 96:45, DOI https://doi.org/10.1007/s00190-022-01631-y.

[14] Torge, W (1980) Geodesy. Walter de Gruyter, Berlin, New York.

[15] Vening Meinesz, FA (1940) Fundamental tables for regional isostatic reduction of gravity values. Publication of the Netherlands Academy of Science, Section 1, DI. 17(3):1–44, URL https://books.google.de/books?id=o5Id6DPXRrgC.

[16] Ye, Z, Tenzer, R and Liu, L (2017) Comparison of spectral and spatial methods for a Moho recovery from gravity and vertical gravity-gradient data. Studia Geophysica et Geodaetica 61:469–496, DOI https://doi.org/10.1007/s11200-016-1049-4.