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AFRGDB_V2.3: The Updated Gravity Database for Africa using RTM Technique

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Abstract *- The IAG sub-commission on gravity and geoid in Africa has the ambitious goal of providing the height reference surface (the geoid) for the continent of Africa. In order to determine the geoid undulations by solving the Stokes integral numerically, the gravity anomalies must be given on a regular regional grid. The available gravity data have large data gaps, particularly at the point gravity on land. The oceanic area is well covered with shipborne data and gravity anomalies derived from altimeter measurements. However, both are naturally measured along tracks. This makes it difficult to estimate a realistic empirical covariance function, which limits the performance of the used least-squares prediction technique. To get rid of this problem, the data is filtered and provided with individual weights. In doing so, the gravity data on land are weighted the highest. The shipborne and altimetry data, on the other hand, are introduced with somewhat less precision. The lowest weight is given to the gravity anomalies computed from the GOCE DIR_R5 geopotential model in order to fill the data gaps. The point gravity data from different sensors are smoothed by applying the widely used RTM reduction scheme. From the smoothed, weighted data, gravity anomalies are predicted on a uniform 5′×5′ grid by applying the weighted least-squares technique. After the final restore step, the AFRGDB (African Gravity Data Base) is obtained in the new version V2.3. Internal and external accuracy investigations are carried out. Finally, an intensive comparison with version V2.0 (AFRGDB_V2.0) is made and discussed.*

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Keywords: Africa, RTM reduction technique, gravity database, geoid determination, weighted least-squares prediction.

1 Introduction

A key objective of the International Association of Geodesy (IAG) Sub-Commission on the gravity and geoid for Africa is to provide the height reference surface for the continent of Africa. This task is elaborated by solving the Stokes integral numerically in space or frequency domain. As a result, the discretized height reference surface is obtained on a grid. This equipotential surface of the gravity field of the Earth, the so-called "geoid", is an important information about the Earth system in many geosciences. In order to be able to solve the Stokes integral numerically, the gravity data must be interpolated from the arbitrarily distributed measuring points onto a regular grid by means of a prediction procedure. The resulting grid covers the entire African continent.

A major challenge is the large amount and extension of data gaps, especially of land-based point gravity (cf. Fig 1). This can yield large errors in the interpolated gridded data. It is well-known that the expected interpolation errors can be counteracted in the best possible way if the supporting data is as smooth as possible. This is achieved by reducing the high-frequency signal components by the RTM reduction. In this paper the RTM smoothed data are used employing the weighted least-squares prediction technique [17] in order to compile the new gravity database AFRGDB_V2.3. It is evaluated against the previous version AFRGDB_V2.0 [11], which was established using the laborious but unambiguous window remove-restore technique with the same input point data.

The available gravity data is briefly explained. They cover the land and oceanic regions in the data window. The interpolation method used is explained. Additional gravity anomalies are synthesized from the GOCE (Gravity field and steady-state Ocean Circulation Explorer) geopotential model DIR_R5. This stabilizes the interpolation in the areas of the mentioned data gaps. The gridded long wavelength component of the gravity anomalies is obtained applying the weighted least-squares prediction technique to the smoothed reference point data. Finally, in the restore step, the gravity anomalies are completed by the short-wavelength parts and residual field components in the grid points. The new gravity data base is thoroughly compared with the previous version AFRGDB_V2.0 [11] on the whole solution window ($40^\circ S \le \phi \le 42^\circ N$, $20^\circ W \le \lambda \le 60^\circ E$). The progress of the creation of this important model under

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the auspices of the IAG is discussed in detail.

2 The Used Data

In this section the gravity data sets available for the gravity and geoid in Africa are described. Additional details can be found in [11]. There are three types of gravity data, namely such on land, shipborne, and altimetry derived gravity anomalies.

2.1 Land Data

Over the past 15 years, 154,037 gravity data points have been made available from various sources for the gravity and geoid for Africa. The first author, head of the sub-commission, is tirelessly active in this central concern. Many data are still not accessible for commercial or political reasons. The authors hope that these data will be made available in the future for the project.

In order to achieve a more homogeneous data distribution, which significantly controls the behaviour of the empirical covariance function, the nearest point gravity value is assigned to a $1'\times1'$ grid cell center [15]. After this filtering process, the remaining homogeneously distributed 127,067 data points are used to determine the parameters of the empirical covariance function.

The filtered land data are then examined for gross errors. The examination process developed for this purpose [5] is based on the least-squares prediction technique [17]. First, a gravity anomaly value from the surrounding data points is predicted for the considered point to be tested. Then, this value is compared with the measured value, and the effect of the current point on the surrounding points is checked. If a gross deviation occurs, the tested point is removed. Figure 1 shows the distribution of the 126,202 gravity points of the land data set after grid filtering and gross-error removal. The existing data gaps are clearly visible and can only be reduced in the short term by releasing existing data, e.g., data from airborne gravimetry. Table 2 comprises the statistical values of the free-air gravity anomalies on land.

2.2 Shipborne Data

After an initial blunder detection procedure of the data [7], the 971,945 gravity values are given. They show a very dense distribution in the North-West part of the data window (cf. Figure 2). The occurring gaps will be filled by the altimetry-derived gravity anomalies. The used blunder detection procedure is similar to the first part of that used for the land data set. However, it is implemented in an iterative way. The threshold for a gross error is set to 1.5 mgal. In order to achieve a more homogeneous data distribution and to reduce the dominance of the shipborne gravity data, they are filtered on a $3'\times3'$ grid. A gross error detection scheme similar to that implemented for land data is adopted to this filtered 148,858 gravity anomalies. Further 184 points are identified as blunders and are eliminated from the data set. Table 2 contains the statistical values of the shipborne free-air gravity anomalies.

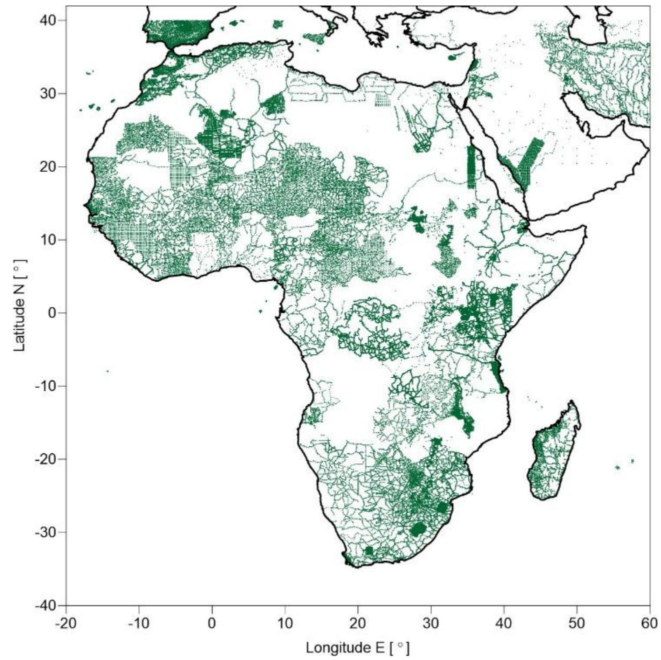

Figure 1. Locations of the land gravity data for Africa.

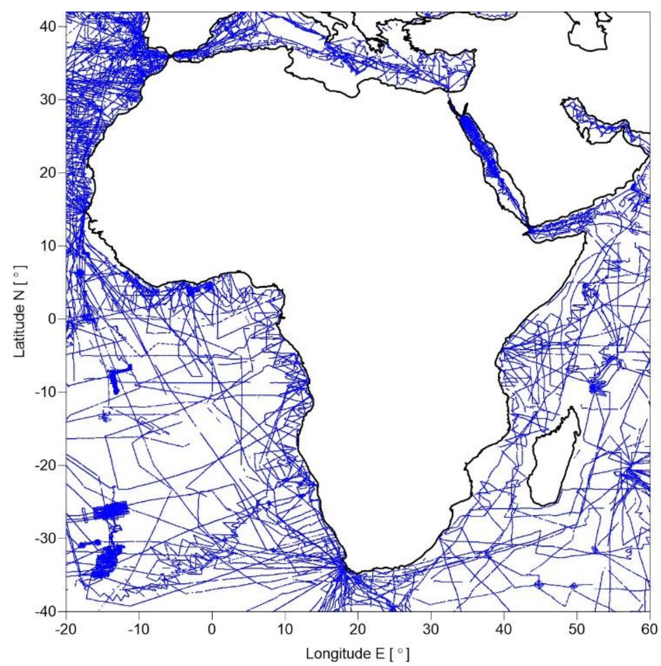

Figure 2. Locations of the shipborne gravity data for Africa.

2.3 Altimetry Data

The open data of 44 repeated cycles of GEOSAT, which is available from the National Geophysical Data Center NGDC (www.ngdc.noaa.gov), are the a priori data set. After a first inspection of the altimetry derived gravity anomalies the averaged tracks are under investigation. This data set consists of 119,249 gravity data points.

The procedure described in section 2.2 when searching for gross errors is adapted to the averaged altimetry data [8]. After the filtering process, a combination with the more precise shipborne data is carried out. The result can be seen in Figure 3. Some gaps along the dominant ship tracks can be inspected, but in general it shows a homogeneous covering with data.

These data follow the ground tracks of GEOSAT. They have been filtered as well on the 3′×3′ grid.

The final gross errors detection described in section 2.1 reduces the altimetry-derived gravity anomalies data set from 70,732 to 70,589 points. Table 2 shows their statistics.

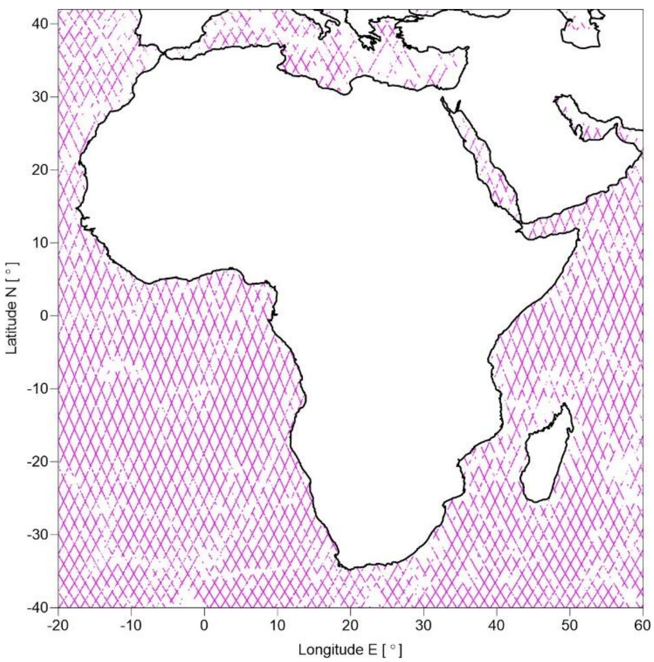

Figure 3. Locations of the altimetry-derived gravity data for Africa.

2.4 Digital Terrain Models

The remove-compute-restore technique, which is used in the present generation of the database AFRGDB_V2.3 in order to smooth the gravity anomalies, requires digital terrain models of different resolutions and smoothness. Especially for this purpose, the following two digital elevation models (DEM) were prepared in the data window (42°S ≤ ϕ ≤ 44°N, 22°W ≤ λ ≤ 62°E) for Africa. The AFH16M03 DEM has a coarse 3′×3′ resolution [10]; it is related to the far field contributions. The AFH16S30 has the fine grid spacing of 30″×30″ (cf. Figure 4). The deepest bathymetric depth is -8291 m and the highest peak is at 5777 m. The gridded heights have a mean value of 1623 m and a standard variation (std) of about 2407 m.

3 Establishment of the AFRGDB_V2.3 Gravity Database for Africa

The procedural method used to create the AFRGDB_V2.3 Gravity Database for Africa is different from the previous versions. It is based on the remove-compute-restore technique. The associated spectral decomposition of the signal information is carried out by means of the residual terrain modeling (RTM), which is explained in the following sections.

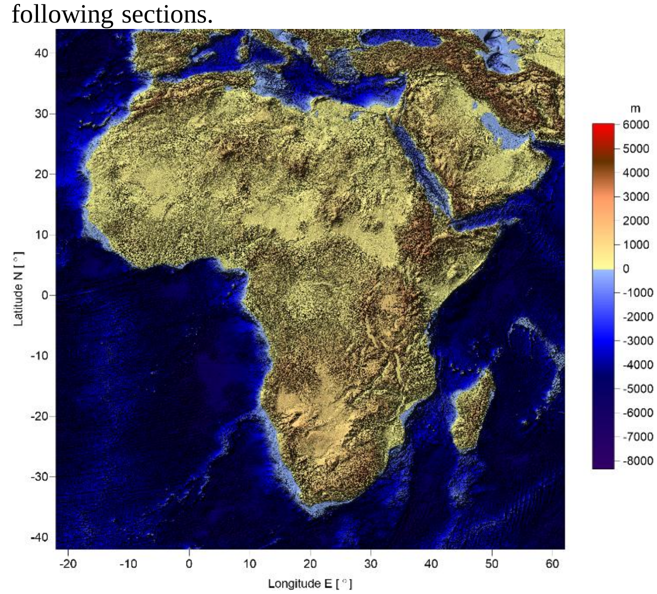

Figure 4. The 30″×30″ AFH16S30 DTM for Africa. Units in [m].

3.1 RTM Technique

In the framework of spectral decomposition, the free air gravity anomalies are separated into three spectral components. Namely, the long wavelength term Δg_{GPM}, the high frequency term, and a remaining residual term Δg_{res}, according to which the relation is rearranged:

$$\Delta g_{res} = \Delta g_F - \Delta g_{GPM} - \Delta g_{RTM}\,. \tag{1}$$

The long wavelength term Δg_{GPM} is modeled by the elected geopotential model. Within this version of the gravity data base for Africa the DIR-R5 is used as GPM which is complete to degree and order $n_{max} = 280$. The high frequency term is attributed to the classical RTM effects on gravity [12]. Residual topographic mass elements, discretized by tesseroids [14], are used up to a spherical distance of R_2 =167 km.

The RTM method requires an appropriate RTM surface, which is suited best to represent the highest frequencies in the used GPM. Therefore a moving average filter is applied to the 3′×3′ DHM for Africa with different filter sizes $n{\times}m$. For all used filter sizes the output RTM surface is generated on a 3′×3′ grids by the moving average filter

$$h_s(i,j) = \frac{1}{n\,m} \sum_{i_1=i-\frac{n-1}{2}}^{i+\frac{n-1}{2}} \sum_{j_1=j-\frac{m-1}{2}}^{j+\frac{m-1}{2}} h(i_1,j_1) \tag{2}$$

where $h_s(i,j)$ is the required smoothed height at the grid pixel (i,j) in the latitude and longitude directions and

$h(i_1, j_1)$ is the height of the used 3′×3′ DHM for Africa of the running pixels. From Eq. (2) it is obvious that both dimensions of the filter n and m should be odd numbers. From Eq. (1) the residual gravity anomalies are computed considering several RTM surfaces (cf. Eq. (2)). As can be concluded from Table 1, the smoothest residual anomalies occur at filter sizes $n \times m$ = 15×15, therefor it is used as RTM surface during the further calculations. The smoothness corresponds to approximately 45′ which is in a good agreement with the resolution of the GPM (180°/280~40′).

Table 1. Statistical parameters of the reduced gravity anomalies Δg_{res} for a test data set, selected by a 30′×30′ filter on the full data set, employing various smoothed DHM reference surfaces

Filter size of the smoothed DHM reference surface	Statistical parameters			
	min.	max.	mean	Std
	mGal	mGal	mGal	mGal
5×5	−122.36	200.32	−0.41	17.47
9×9	−115.79	148.05	−0.21	15.41
13×13	−112.43	118.27	−0.12	14.28
15×15	**−111.71**	**118.11**	**−0.10**	**14.13**
17×17	−111.92	117.97	−0.09	14.23
19×19	−113.46	117.62	−0.09	14.54
23×23	−119.95	116.78	−0.10	15.67

In order to reduce the influence of the large data gaps, in particular in the land gravity data, a supporting data grid with pseudo-observations, synthetically generated from the GPM is applied:

$$\Delta g_F = \Delta g_{GPM} \tag{3}$$

For the so-called "underlying grid", the residual gravity anomalies follow from inserting Eq. (3) into Eq. (1). We get the simple expression:

$$\Delta g_{res} = -\Delta g_{RTM} \, . \tag{4}$$

3.2 Gravity Reduction

Equation (1) is used to compute the reduced anomalies. The mass effects on the RTM contributions Δg_{RTM} are evaluated up to the spherical distance $R_2 = 167$ km. A modified version of the TC-program [12] was used to compute the RTM effects [4]. The long-wavelength part Δg_{GPM} is computed with the routine GRVHRM [2] from the GOCE GPM DIR_R5 with the maximum degree $n_{max} = 280$. Table 2 presents the statistics of the free-air and the residual anomalies for each gravity data set described in Section 2.

From the respective statistical parameters listed in Table 2 the following comparisons between the free-air and the RTM-reduced gravity anomalies can be concluded:

- Compared to free-air, Δg_{RTM} are more centered,
- Their standard deviation are smaller by a factor of 2-3 (except for the altimetry-derived data),
- The statistics of the RTM-reduced anomalies for the total data and for the underlying grid match very well. This is a clear indication that the given gravity data are well screened and the RTM has a significant impact on smoothing.

Table 2. Statistics of the free-air, the RTM-reduced, the final gridded RTM-reduces and free-air gravity anomalies. Units are in [mGal]

Reduction technique	Data region	Statistical parameters			
		min	max	mean	std
Free-air (remove)	Land Shipborne Altimetry	−163.20 −238.30 −172.23	465.50 354.40 156.60	9.84 −6.21 4.09	40.93 34.90 18.23
	Total	−238.30	465.50	1.76	35.44
	Underlying	−177.30	186.05	3.26	27.21
RTM-reduced (remove)	Land Shipborne Altimetry	−93.35 −140.15 −111.87	140.69 118.37 92.01	0.10 −1.26 5.69	14.37 15.50 12.10
	Total	**−140.15**	**140.69**	**0.66**	**14.68**
	Underlying	**−282.80**	**146.79**	**−0.06**	**14.75**
gridded RTM-reduced	Total	-137.64	163.55	0.61	11.20
gridded **final free-air** (restore)	**Total** **AFRGDB_V2.3**	**-238.18**	**511.14**	**3.19**	**31.84**
windows technique	**Total AFRGDB_V2.0**	**-243.04**	**468.00**	**3.04**	**31.94**
	AFRGDB_V2.3 – AFRGDB_V2.0	**-156.30**	**554.36**	**0.15**	**9.78**

3.3 Interpolation

To combine the prepared data sets on a final grid of 5′×5′ resolution covering the African window (40°S ≤ ϕ ≤ 42°N, 20°W ≤ λ ≤ 60°E), a weighted least-squares prediction technique has been applied to the respective RTM-reduced gravity anomalies Δg_{res}. The carefully selected used standard deviations for the four gravity anomaly types are:

- Land data: σ_{land} = 1 mGal
- Shipborne data: $\sigma_{shipborne}$ = 3 mGal
- Altimetry data: $\sigma_{Altimetry}$ = 5 mGal
- Underlying grid: $\sigma_{underlying\ grid}$ = 20 mGal.

The used least-squares interpolation technique employs the generalized covariance model of Hirvonen [17]

$$C(s) = \frac{C_o}{(1+A^2 s^2)^p} \quad . \tag{5}$$

The spherical distance between the pair of points under consideration is denoted by s. The parameter A is defined as

$$A = \frac{1}{\xi}\left(2^{\frac{1}{p}} - 1\right)^{\frac{1}{2}} \tag{6}$$

The variance C_o in Eq. (5) and the correlation length ξ

in Eq. (6) are empirically determined from the data. The dimensionless curvature parameter χ is related to the curvature κ of the covariance function at $s = 0$ by [15]

$$\chi = \frac{\kappa \xi^2}{C_o} \quad (7)$$

Therefore, it affects the values of the covariance function $C(s)$ near the computation point. The curvature parameter χ is related to the parameter p as [16]; [1]

$$\chi = 2p\left(2^{\frac{1}{p}} - 1\right) \quad (8)$$

A least-squares regression technique has been adopted by [6] to best estimate the parameter p in the process of the empirical covariance function fitting. The estimated parameters defining the used covariance function for the combination of the four data sets are listed in Table 3.

Table 3. Empirically determined parameters defining the covariance function

Parameter	value	unit
C_o	215.62	$mGal^2$
ξ	24.78	km
p	6.222	—

Figure 5 shows the empirical and the modelled covariance functions. The latter is estimated, as mentioned above, by the least-squares fitting technique developed by [6]. The marvelous representation of the empirical covariance values by the adjusted modelled covariance function is obvious.

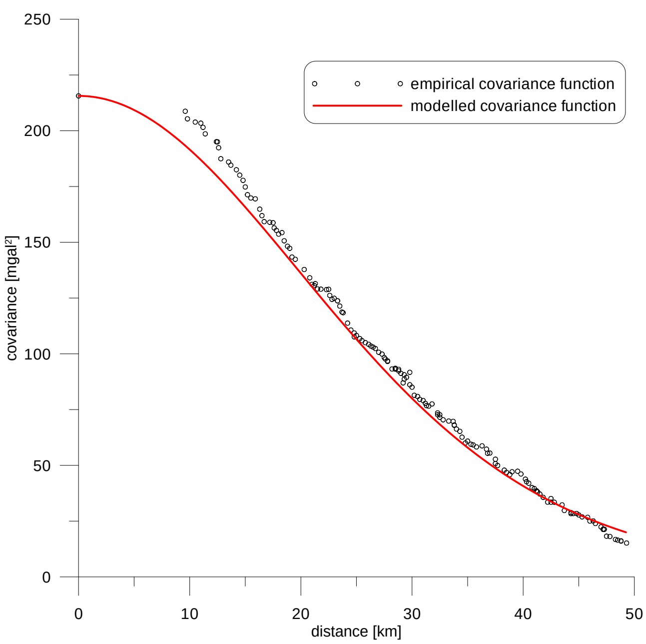

Figure 5. The empirical and modelled covariance functions.

Figure 6 illustrates the 5′×5′ interpolated GPM-reduced and RTM-smoothed gravity anomalies Δg^G_{res} for Africa. Table 2 shows the statistical parameters of the gridded RTM-reduced anomalies.

3.4 Restore

Finally, the three anomaly parts are restored on a 5′×5′ grid. This restore step is described by the following expression, which looks similar to Eq. (1), but Eq. (1) represents the gravity information on the gravity data points:

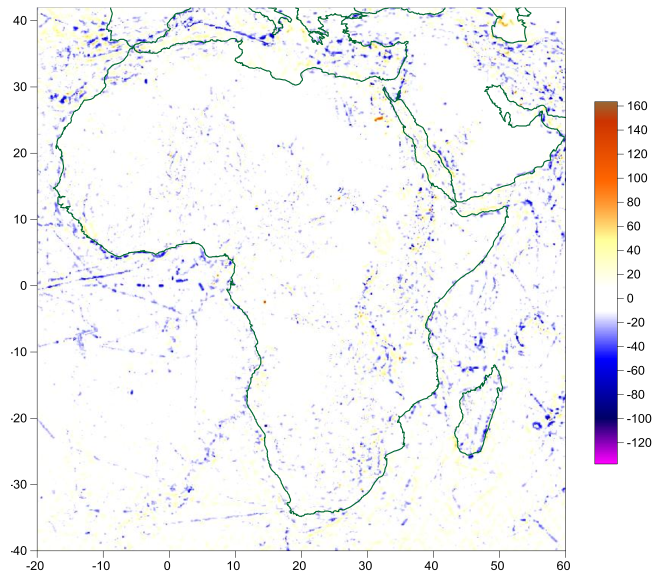

Figure 6. The 5′×5′ interpolated RTM-reduced gravity anomalies Δg^G_{res} for Africa. Units in [mGal].

$$\Delta g^G_F = \Delta g^G_{res} + \Delta g^G_{RTM} + \Delta g^G_{GPM}. \quad (9)$$

To distinguish between the anomalies in the given data points and the final grid points, the latter is indicated with the superscript G.

The restored free-air gravity anomaly Δg^G_F elaborated by Eq. (9) correspond to the values for the new AFRGDB_V2.3 gravity database for Africa.

Figure 7 illustrates the 5′×5′ African free-air gravity anomaly database AFRGDB_V2.3. Its statistical values are listed in Table 2.

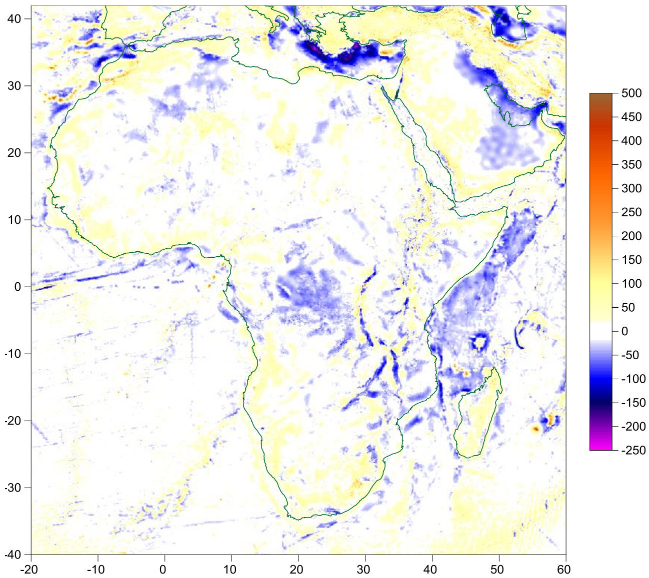

Figure 7. The updated AFRGDB_V2.3 free-air gravity anomaly database for Africa. Units in [mGal].

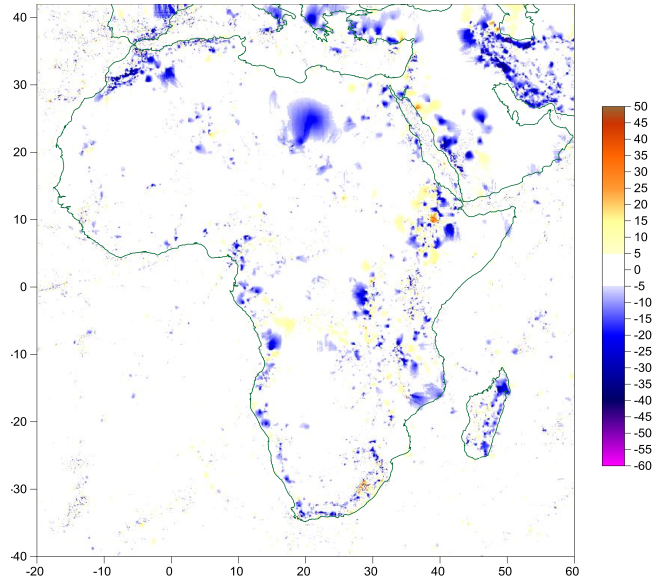

Figure 8. Internal validation of the AFRGDB_V2.3 gravity database in terms of residuals at the data points. Units in [mGal].

4 Validation

The new AFRGDB_V2.3 gravity database for Africa has been validated. It is validated on the one hand based on the given point values (internal validation) and on the other hand in relation to unused point values (external validation). The corresponding procedure and the result of the validation are presented in the following sections.

4.1 Internal Validation

The internal validation is based on the observation residuals. All the observations are the point values shown in Figs. 1–3. This means nothing but the difference between the point gravity anomalies and the interpolated respective values from the AFRGDB_V2.3 gravity database at the location of the data points. The residuals are plotted in Fig. 8. The statistical parameters are listed in Table 4. The white areas in Fig. 8 represent 81.34% of the points having absolute residuals below 5 mGal.

The histogram of the absolute residuals, defined as data minus database values, is drawn in Fig. 9. The histogram of the residuals shows a Gaussian normal distribution with high precision index, which assures relatively high precision of the created gravity database. 84.17% of the data points have residuals smaller than the standard deviation.

4.2 External Validation

Data points that have not used to build the AFRGDB_V2.3 gravity database due to the grid filtering process were used as external validation of the quality of the database being built. This huge data set comprises around 27 thousands and 871 thousands points on land and on sea, respectively (cf. Fig. 10).

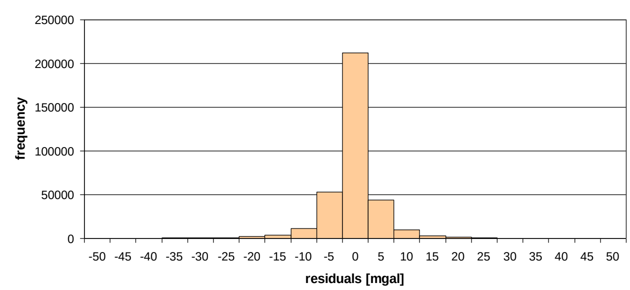

Figure 9. Histogram of the residuals at data points (internal validation) for the AFRGDB_V2.3 gravity database.

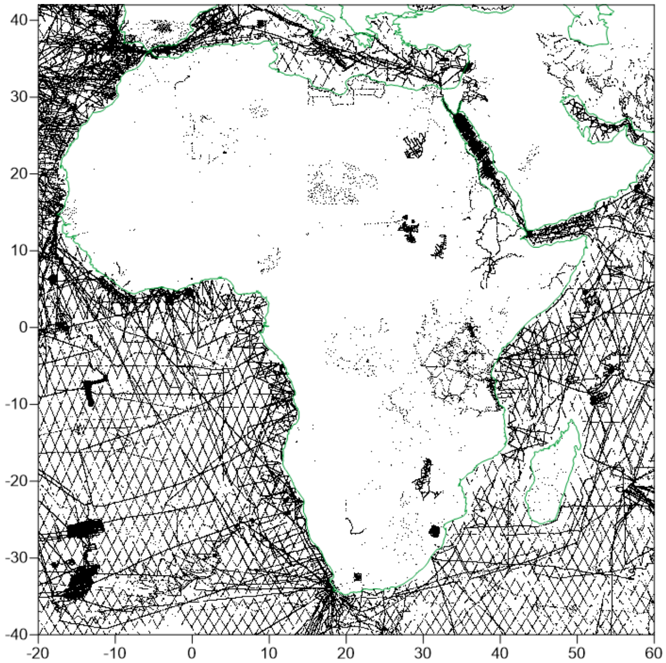

Figure 10. External verification point distribution for the AFRGDB_V2.3 gravity database.

The residuals at the external verification points are plotted in Fig. 11. The statistical values of the differences between the data and the values of the AFRGDB_V2.3 are given in Table 4. The white pattern in Fig. 11 represents absolute residuals smaller than 5 mGal (69.18% of the points).

Examining Fig. 10 shows clearly that the distribution of existing points available for the external check on land is quite sparse and huge data gaps exist. This is the reason of the artificial patterns seen in Fig. 11, which are nothing but false interpolated large residuals resulting from the Kriging gridding technique used to prepare Fig. 11. This, by the way, proofs the advantage of the weighted least-squared prediction technique adopted in the current investigation (no artificial false patterns appearing in Figs. 6 and 7).

The histogram of the absolute residuals at the points available for the external check is drawn in Fig. 12. The histogram shows a Gaussian normal distribution with high precision index assuring relatively high precision of the created gravity database. 80.09% of the data points have residuals below the standard deviation.

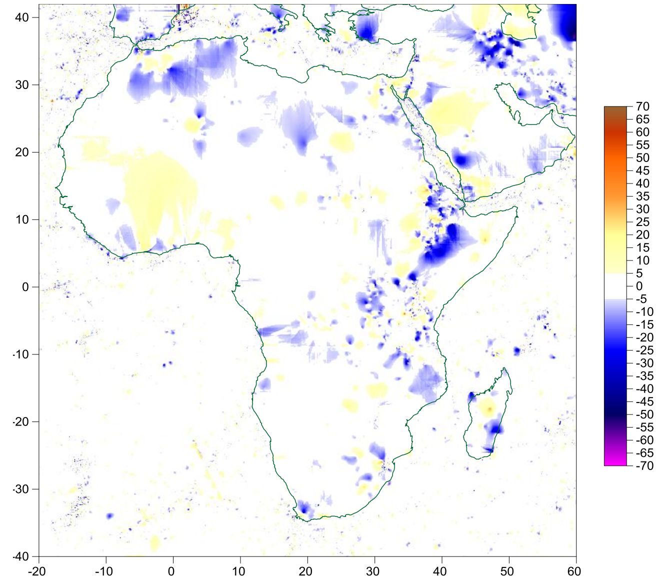

Figure 11. Residuals at the external check points for the AFRGDB_V2.3 gravity database. Units in [mGal].

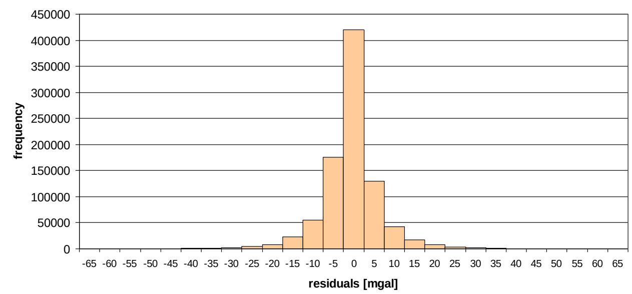

Figure 12. Histogram of the residuals at the external check points for the AFRGDB_V2.3 gravity database.

5 Comparison with the Previous Gravity Database

The previous AFRGDB_V2.0 gravity database was generated using the same point gravity anomalies as in the current AFRGDB_V2.3 model, but with the window remove-restore technique [2]; [3]. As GPM the EIGEN-6C4 complete to d/o $n_{max} = 2190$ [13] was used, which is a combined model of terrestrial gravity anomalies and satellite observations. More details about the creation of the previous AFRGDB_V2.0 gravity database can be found in [11]. Figure 13 illustrates the free-air gravity anomalies of the AFRGDB_V2.0 gravity database. Their statistical parameters of the respective free-air anomalies are listed in Table 2.

The difference between the old and new gravity databases for Africa (AFRGDB_V2.3 – AFRGDB_V2.0) is shown in Fig. 14. No tilting parameters are applied. The statistical measures are given in Table 2. Most absolute differences are below 10 mGal as indicated by the white color appearing in Fig. 14. The larger difference are restricted to the high mountainous areas.

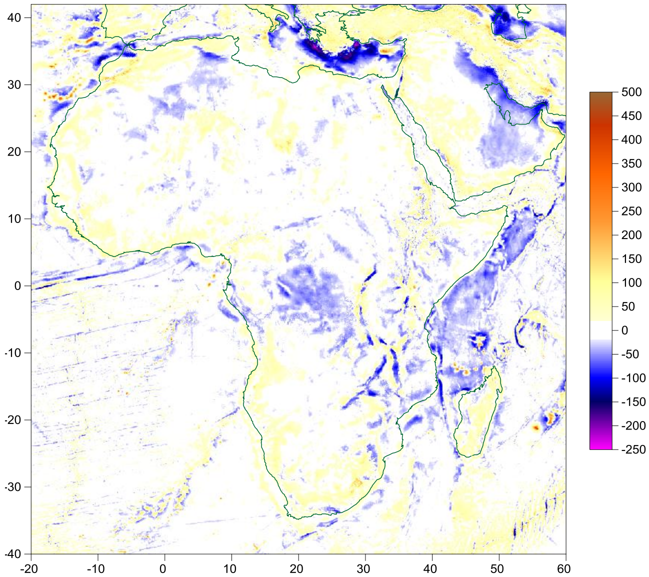

Figure 13. The previous gravity database for Africa (AFRGDB_V2.0). Units in [mGal].

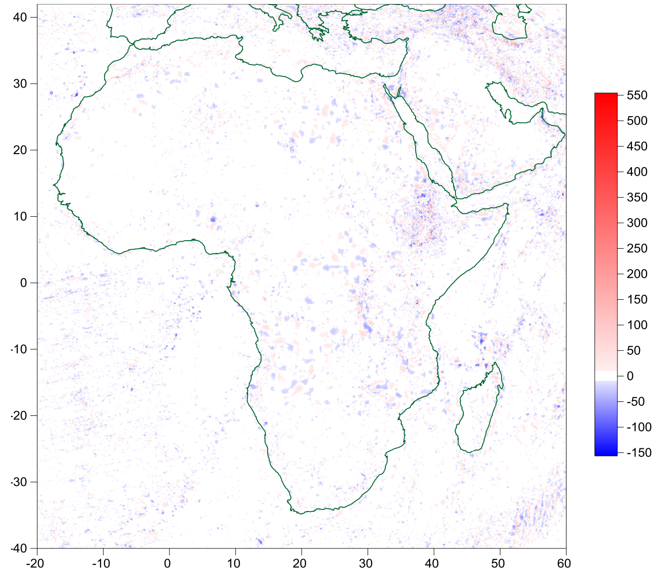

Figure 14. Difference between the old and new gravity databases for Africa (AFRGDB_V2.3 – AFRGDB_V2.0). Units in [mGal].

Table 4 lists the statistical parameters for the internal and external checks of the old and new gravity databases for Africa. It obviously declares that both models are quite comparable in-spite-of the fact that the used methodology to build both models are fairly different. This shows that the establishment strategy of the African gravity database becomes vigorous to some extent.

Table 4. Statistical parameters for the internal and external checks of the old and new gravity databases for Africa

Type of validation	Region	AFRGDB_V2.3				AFRGDB_V2.0			
		min.	max.	mean	std	min.	max.	mean	std
		mGal	mGal	mGal	mGal	mGal	mGal	mGal	mGal
Internal	Land	−51.04	51.05	−1.03	7.50	−50.80	50.88	−1.00	7.56
	Sea	−40.88	61.94	0.04	4.21	−43.90	55.71	−0.01	3.94
	Total	−51.04	61.94	−0.35	5.67	−50.80	55.71	−0.37	5.56
External	Land	−65.25	65.09	−0.72	9.45	−66.97	67.25	−0.76	9.62
	Sea	−50.43	50.43	−0.55	7.20	−48.75	48.62	−0.58	6.89
	Total	−65.25	65.09	−0.55	7.28	−66.97	67.25	−0.59	6.99

6 Conclusion

A new version of the African gravity database has been built-up where the RTM method was applied. In addition to the available point gravity data on land and ocean, a grid was added to close the data gaps, in particular on land, with reasonable data. This underlying grid has a resolution of 15′×15′ and was generated from the GOCE Dir_R5 GPM by synthesis up to d/o $n_{max} = 280$. The point data on land have been filtered to a grid of 1′×1′ resolution, while the data over the ocean were filtered on a grid with 3′×3′ resolution in order to decrease the number of data along the satellite ground tracks and ship tracks. By filtering the point data to an equidistant grid, the behavior of the covariance function should also be improved, especially for the covariance in the vicinity of the calculation point.

The method of spectral decomposition was used, which results in a division into a long wavelength, short wavelength and residual contributions. The gravity anomalies of long wavelength are again generated from the GOCE Dir_R5 GPM by synthesis up to d/o $n_{max} = 280$. The short wavelength part was attributed to the residual terrain. The final joining of all data is carried out by the weighted least-squares prediction technique. The new AFRGDB_V2.3 gravity database for Africa is discretized on a 5′×5′ and has an internal precision of about 5.6 mGal.

The AFRGDB_V2.3 gravity database was compared to the previous AFRGDB_V2.0 gravity database. Both models, which were created using different methods, show a very good agreement with external accuracy of about 7 mGal. This makes it clear that the IAG Sub-Commission on gravity and geoid in Africa has already developed robust methods and will continue to work on them. The comparison also declares that the RTM method has significant advantages in terms of the required CPU time.

Of course, the new AFRGDB_V2.3 gravity database can not only be used for a corresponding calculation update of the geoid model. Free-air gravity anomalies reveal interesting geophysical signals which are of interest to all Earth system sciences.

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